HOW TO BECOME QUICK AT FIGURES.

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—COMPRISING—

THE SHORTEST, QUICKEST, AND BEST METHODS OF BUSINESS CALCULATIONS.

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MEN WANTED.

"It has been truly said that the great want of the age is men. Men of thought; men of action. Men who are not for sale. Men who are honest to the heart's core. Men who will condemn wrong in friend or foe — in themselves as well as others. Men whose consciences are as steady as the needle to the pole. Men who will stand for the right if the heavens totter and the earth reels. Men who can tell the truth and look the world and the devil right in the eye. Men who neither swagger nor flinch. *Men who are Quick at Figures.* Men who can have courage without whistling for it, and joy without shouting to bring it. Men through whom the current of everlasting life runs still, and deep and strong. Men too large for certain limits, and too strong for sectarian bands. Men who know their message and tell it. Men who know their duty and do it. Men who know their place and fill it. Men who mind their own business. Men who will not lie. Men who are not too lazy to work, nor too proud to be poor. When in office, the workshop, in the counting-room, in the bank, in every place of trust and responsibility, we can have such men as these, we shall have a christian civilization — the highest and best the world ever saw."

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CONTENTS — CONTINUED.

DO SOMETHING.

Do not spend your precious time in wishing, and watching, and waiting for something to turn up. If you do, you may wish and watch and wait forever. You can do it if you wish, but you must put forth the effort. Idleness and indifference never accomplished anything. It takes energy and push to make headway in the world, and an active, energetic, persevering man is sure to succeed. If he can not do one thing he will do something else. If he can not succeed in one direction he will in some other. He will do something. He will not waste his time in idleness. There is no lack of work, no lack of opportunities. Do what comes to your hand, and do it well. True progress is from the less to the greater. You must begin low if you would build high. Work is ordinarily the measure of success. Quit resolving and re-resolving and go and do something. — *School Supplement.*

ADDITION.

The adding of one or more columns of figures should be done without mental labor, and may be acquired by anyone with a good deal of practice.

The art of adding quickly is acquired by learning to *read* a column of figures as you would a sentence composed of words, and those words composed of letters. By *Practice* we have become so familiar with letters that when we see them grouped together, it is unnecessary to separate them, or spell out the words, but we can tell at a glance what the word is.

By *Practice* we may become so familiar with figures that when we see a group of them, we can tell at a glance what the sum of them reads, without spelling the figures at all.

In practicing the reading of a column of figures in this way, we do not let the brain work at all, but simply pass the eye over the figures (see drill tables) as if you were reading a sentence, slowly at first, but increase the speed as proficiency is acquired.

A few minutes daily practice will produce astonishing results in a very short time; beginning with two figures, then three, four, and so on until finally we become able to write the *Sum* total of long columns. For example, when we see the figures 9, 8, 6, 4, we know at a glance that the sum is 27 without reading the figures themselves or spelling them out.

Reading a column of figures as the reading of a sentence, is done by dividing a large group of figures

into smaller ones and from group to group through the column, just as from word to word we read through a sentence.

We give various methods, but commend the *grouping* method as the best and most practiced.

Addition is more frequently used than all other operations combined.

The most important qualities of an accountant are accuracy and speed. The most speedy calculators are usually the most correct.

It is a deplorable fact that not one in one hundred of our graduates fresh from the High School can add a column of figures correctly without many trials.

No labor should be regarded too great to master this, the key to all numerical as well as business transactions.

RESULTS ONLY.

Never spell your way through a column, thus: 6 and 8 are 14, and 9 are 23, etc. It is just as easy to name results only, and much more rapid.

For the purpose of explaining a method, examples will be sometimes spelled out, but it is never recommended as a method to be adopted.

```
 3 6 7
 8 5 4
 9 7 6
 3 8 9
 7 3 6
 5 4 8
 4 7 9
 7 2 6
 6 9 8
-------
5 7 7 3
```

Begin at the bottom of the right hand column, and name results only:

14, 23, 31, 37, 46, 52, 56, 63.

Then adding 6, the carrying figure, to the second column, we have:

15, 17, 24, 28, 31, 39, 46, 51, 57.

Again carrying 5 to the next column we say:

11, 18, 22, 27, 34, 37, 46, 54, 57,

which completes the operation.

NOTE.—When 9 occurs in addition it is easier to add ten and subract one mentally. thus: instead of 9 + 8=17, say, 10 + 8=18—1=17.

GROUPING.

487 943 345 868 145 989 568 673 ——— 5018	SPELLING PROCESS. $11+10+4+10+3+10=48$ $4+10+3+10+2+10+10+2=51$ $5+10+1+10+10+1+10+3=50$

The sum of any two figures can never exceed a *ten;* therefore, the left hand figure of the sum of any two figures must always be *one.*

It is, by this method, no more difficult to read the sum of two figures, than to read a single figure.

In the process we pay no attention to the figures as they stand written, but only the sum of two of them.

In the above example, instead of reading 11, 20, 25, 33, etc., we say:

11, 21, 25, 35, 38, 48.
4, 14, 17, 27, 29, 39, 49, 51.
5, 15, 16, 26, 36, 37, 47, 50.

It will be noticed that the left hand figure increases by regular notation, 1, 2, 3, 4, 5, 6, etc.

Practice will enable one to add much more rapidly than by the old method, with less liability to error, because it is systematic and simple, and requires less mental labor.

GROUPING.

By this method we mentally group the figures above 10 and under 20.

To the first group add the tens of the next group, and to this sum add the units of the second group.

In order to become proficient in grouping, familiarity of totals will be absolutely necessary. The *sum* of the figures must be read instead of the figures themselves. By a proper understanding of this method, the mental labor is much less than by the old ones; half the labor is simply counting by regular notation, because the left hand or tens figure to be added is always the same, and the right hand or units figure usually of a small denomination. By an analysis of the example under this head, we find the first column in group to read:—

$$\begin{array}{l}\left.\begin{matrix}7\\3\end{matrix}\right\}10\\ \left.\begin{matrix}5\\8\end{matrix}\right\}13\\ \left.\begin{matrix}5\\9\end{matrix}\right\}14\\ \left.\begin{matrix}8\\3\end{matrix}\right\}11\end{array}$$

To 11, the first group, add the tens of the second group, we have 21, adding the unit 4 of this group we have 25, adding the tens of the third group we have 35, and adding 3, the units figure of the group we have 38, adding the tens of the fourth group we have 48 which completes the operation.

SECOND COLUMN.

$$\begin{array}{l}\left.\begin{matrix}8\\4\end{matrix}\right\}12\\ \left.\begin{matrix}4\\6\end{matrix}\right\}10\\ \left.\begin{matrix}4\\8\end{matrix}\right\}12\\ \left.\begin{matrix}6\\7\end{matrix}\right\}13\end{array}$$

4 the carrying figure of the previous column, added to the first group we have:—

$4+10+3+10+2+10+10+2=51$

By naming result only, we have:—

4, 14, 17, 27, 29, 39, 49, 51

We recommend to name the carrying figure the first, and when a nine occurs call it 10, and subtract 1 in the operation.

DRILL TABLES.

These Drill Tables must be thoroughly mastered. Read from left to right and right to left, the sum *only*, as rapidly as possible ; when you falter or make an error, go back and start again.

We would advise teachers to have a daily blackboard exercise of these drill tables for addition.

Divide the exercises. Write series that are simple and easily comprehended.

1st. Read the sum of two figures only, in every possible combination.

2d. Read the sum of three figures only.

3d. Read in two columns of two figures, then three, and also mixed numbers.

It will make a pleasant, entertaining, as well as useful change from the regular routine. One month's daily exercise will result in surprising proficiency.

The whole school should be engaged in these exercises, and combinations varied and made more difficult, slowly.

Hint. One of the most accomplished accountants in the city of Boston, informed the writer that he owed his success as an accountant entirely to his being a *confident* adder, which he acquired largely by appropriating all figures in sight, and reading their sum only. Numbers on the streets, numbers of railroad cars, etc., etc., served as drill masters to him on many occasions.

DRILL TABLE No. 1.

3	4	7	4	8	4	4	8	2	3	5
2	5	6	3	4	9	5	3	4	4	6
—	—	—	—	—	—	—	—	—	—	—
8	6	5	8	4	6	7	5	8	4	7
7	7	8	4	9	5	8	9	7	9	9
—	—	—	—	—	—	—	—	—	—	—
8	4	3	9	0	4	6	7	8	5	6
3	4	5	6	7	8	9	4	5	6	7
—	—	—	—	—	—	—	—	—	—	—
9	8	7	6	4	5	7	8	4	6	9
9	6	8	9	8	4	9	7	4	0	7
—	—	—	—	—	—	—	—	—	—	—
7	8	9	8	7	9	9	6	7	5	6
8	9	7	9	8	3	1	0	3	2	8
—	—	—	—	—	—	—	—	—	—	—
3	8	4	3	9	4	6	9	8	4	5
9	9	8	9	8	4	3	6	5	9	0
—	—	—	—	—	—	—	—	—	—	—
2	8	7	6	5	3	5	7	9	3	1
8	7	5	7	6	5	9	8	2	8	5
—	—	—	—	—	—	—	—	—	—	—
5	8	2	9	7	6	3	4	7	8	8
8	9	7	8	5	7	8	9	4	3	7
—	—	—	—	—	—	—	—	—	—	—
3	8	9	7	6	5	1	2	4	6	8
7	5	3	3	9	5	9	7	6	5	8
—	—	—	—	—	—	—	—	—	—	—

DRILL TABLE No. 2.

3	4	7	4	8	4	4	8	2	3	5
2	5	6	3	4	9	5	3	4	4	6
8	6	5	8	4	6	7	5	8	4	7
—	—	—	—	—	—	—	—	—	—	—

7	7	8	4	9	5	8	9	7	9	9
8	4	3	9	0	4	6	7	8	5	6
3	4	5	6	7	8	9	4	5	6	7
—	—	—	—	—	—	—	—	—	—	—

9	8	7	6	4	5	7	8	4	6	9
7	8	9	8	7	9	9	6	7	5	6
8	9	7	9	8	3	1	0	3	2	8
—	—	—	—	—	—	—	—	—	—	—

9	6	8	9	8	4	9	7	4	0	7
3	8	4	3	9	4	6	9	8	4	5
9	9	8	9	8	4	3	6	5	9	0
—	—	—	—	—	—	—	—	—	—	—

5	8	2	9	7	6	3	4	7	8	8
2	8	7	6	5	3	5	7	9	3	1
8	7	5	7	6	5	9	8	2	8	5
—	—	—	—	—	—	—	—	—	—	—

8	9	7	8	5	7	8	9	4	3	7
3	8	9	7	6	5	1	2	4	6	8
7	5	3	3	9	5	9	7	6	5	8
—	—	—	—	—	—	—	—	—	—	—

2	8	7	3	2	8	8	6	6	5	4
7	8	7	9	7	9	3	5	9	7	9
8	9	3	2	5	4	7	6	8	0	5
—	—	—	—	—	—	—	—	—	—	—

DRILL TABLE No. 3.

3	4	7	4	8	4	4	8	2	3	5
2	5	6	3	4	9	5	3	4	4	6
8	6	5	8	4	6	7	5	8	4	7
7	7	8	4	9	5	8	9	7	9	9

8	4	3	9	0	4	6	7	8	5	6
9	8	7	6	4	5	7	8	4	6	9
7	8	9	8	7	9	9	6	7	5	6
8	9	7	9	8	3	1	0	3	2	8

3	4	5	6	7	8	9	4	5	6	7
9	6	8	9	8	4	9	7	4	0	7
3	8	4	3	9	4	6	9	8	4	5
9	9	8	9	8	4	3	6	5	9	0

8	9	7	8	5	7	8	9	4	3	7
5	8	2	9	7	6	3	4	7	8	8
2	8	7	6	5	3	5	7	9	3	1
8	7	5	7	6	5	9	8	2	8	5

2	7	2	7	1	2	3	4	5	6	7
7	7	6	5	4	3	2	1	2	3	4
0	9	5	7	3	7	9	1	5	5	9
7	5	3	3	9	5	9	7	6	5	8

3	8	9	7	6	5	1	2	4	6	8
2	8	7	3	2	8	8	6	6	5	4
7	8	7	9	7	9	3	5	9	7	9
8	9	3	2	5	4	7	6	8	0	5

TWO COLUMNS AT ONE OPERATION,

FROM LEFT TO RIGHT

If two or more columns can be added at one operation, there must be some rule or method by which it is done. The following illustrates one of the most practical methods :—

	SPELLING PROCESS.
25	
63	$27 + 50 = 77 + 4 = 81$
78	$81 + 30 = 111 + 4 = 115$
34	$115 + 70 = 185 + 8 = 193$
54	$193 + 60 = 253 + 3 = 256$
27	$256 + 20 = 276 + 5 = 281$
281	

This process consists simply in adding the tens first and then the units.

READING PROCESS.

27, 77, 81, 115, 185, 193, 253, 256, 276, 281.

Practice will enable one to add two or more columns without much mental effort, because it is just as easy to say, $27 + 50 = 77 + 4 = 81$, as it is to say, $5 + 2 = 7$, or $7 + 4 = 11$.

Proficiency in the above method will soon enable one to add without separating each number into tens and units, thus :—

27, 81, 115, 193, 256, 281.

The all important thing is familiarity of totals, and we herewith append drill tables, applicable to this kind of addition, which must be read from each direction as advised before.

DRILL TABLE No. 4.

24	43	52	84	12
32	25	34	21	32
—	—	—	—	—
25	64	25	72	34
47	52	38	29	63
—	—	—	—	—
23	63	89	38	85
38	94	98	93	04
—	—	—	—	—
95	38	68	94	39
64	46	89	38	85
—	—	—	—	—
25	93	38	97	39
78	36	48	68	95
—	—	—	—	—
98	97	39	76	84
38	84	98	34	38
—	—	—	—	—
89	78	63	82	21
99	87	36	67	79
—	—	—	—	—
75	85	95	65	55
81	98	79	56	29
—	—	—	—	—
76	63	55	78	89
86	81	75	84	92
—	—	—	—	—

ADDITION.

DRILL TABLES.

Read rapidly from left to right and right to left the sum only in the following combinations: —

9	8	7	8	9	6	7	8	9	8	9	7	9	8	9
8	7	9	4	6	8	3	4	3	6	5	8	5	4	3

24	35	72	94	39	36	28
32	46	82	38	48	59	39

9	1	9	9	6	2	8	6	7	8	5	8	9
5	2	8	6	5	3	4	7	6	3	6	5	8
4	6	3	3	9	8	3	8	5	2	9	6	7

24	26	35	53	63	36	37	73	83
12	36	45	54	64	46	47	74	84
32	46	55	55	65	56	57	75	85

1.25	12.50	9·50	29.40	45.90
.84	3.42	.21	3.50	1.50
5.62	.40	4.21	6.30	56.89

Combinations should be varied and made more difficult as proficiency is acquired. The secret in adding rapidly consists in familiarity with totals of combinations. Counting is not adding, and spelling is not reading.

THREE COLUMNS AT ONE OPERATION,

FROM RIGHT TO LEFT.

Three or more columns may be added at one operation thus :—

$$\begin{array}{r} 223 \\ 425 \\ 384 \\ 256 \\ \hline 1288 \end{array}$$

OPERATION.—256+4=260, 260+80=340, 340+300=640, 640+5=645, 645+20=665, 665+400=1065, 1065+3=1068, 1068+20=1088, 1088+200=1288.

By naming results only, we have :—

260, 340, 640,
645, 665, 1065,
1068, 1088, 1288.

It will be noticed that beyond two, or at most three, columns wide and a few columns deep, this method requires more mental labor than the previous ones, and is not considered very practical except for development of the mental faculties.

EXAMPLE 2.

$$\begin{array}{r} 2.25 \\ .35 \\ 8.46 \\ .08 \\ 15.60 \\ 2.13 \\ 1.40 \\ \hline 30.27 \end{array}$$

PROCESS.

340, 350, 353, 1353, 1853, 1913, 1921,
2721, 2761, 2767, 2797, 2802,
3002, 3022, 3027.

THE LIGHTNING METHOD.

AN OLD TRICK EXPOSED.

This simple and wonderful combination of figures has deluded many into believing that adding from left to right such a large body of figures, instantaneously and correctly, to be a Herculean task.

Large sums have been paid by the unsuspecting and credulous for the possession of the wonderful secret.

It is probably the simplest, as well as the most delusive combination of numbers, because if operated in the hands of an expert it is almost impossible to be detected, unless by those who know the secret.

EXAMPLE :

	3	4	5	6	8	8	9	3	4	2
	6	5	4	3	1	1	0	6	5	7
	3	5	8	4	3	8	4	3	8	9
	6	4	1	5	6	1	5	6	1	0
	7	8	6	8	9	8	4	5	6	8
2	7	8	6	8	9	8	4	5	6	6

The operation is as follows: A line of figures is given you, to the length of which you are wholly indifferent, as in above example.

3 4 5 6 8 8 9 3 4 2 (First line.)

Now you claim the privilege of writing a line immediately underneath it. Your line will be in pairs of 9's, and will read:

6 5 4 3 1 1 0 6 5 7 (Second line.)

Now ask another line immediately under these; any figures whatever, for example:

3 5 8 4 3 8 4 3 8 9 (Third Line.)

You write another to pair it into 9's:

6 4 1 5 6 1 5 6 1 0 (Fourth Line.)

Another line of figures is given you:

7 8 6 8 9 8 4 5 6 8 Fifth or Key Line.

This is called the key line, because the sum of the entire column is simply this line repeated, with 2 subtracted from the right, and 2 annexed to the left. In other words, the result being just like it, except the units' place, which is as many less than the units in the key line as there are pairs of lines, and annex a similar number to the extreme left of the sum. The number of lines is necessarily odd.

For the purpose of explanation we will add the column without the key line.

	3	4	5	6	8	8	9	3	4	2
	6	5	4	3	1	1	0	6	5	7
	3	5	8	4	3	8	4	3	8	9
	6	4	1	5	6	1	5	6	1	0
1	9	9	9	9	9	9	9	9	9	8
										2
2	0	0	0	0	0	0	0	0	0	0

Subtracting two from the right hand figure of the key line is equivalent to adding two to the other columns, which would bring all the other figures ciphers as above.

It matters not, therefore, what the key line is, so long as we call the right hand figure two less, and annex two to the left.

A little practice will make this a ve`y interesting trick.

SECOND APPLICATION OF THE "LIGHTNING" METHOD.

Take any three columns of figures as follows:—

6 3 4 5 6
3 8 4 2 9
2 5 6 3 6

Pair the last two with 9's and all the columns will stand thus:—

$$\begin{array}{r}
6\,3\,4\,5\,6\\
3\,8\,4\,2\,9\\
6\,1\,5\,7\,0\\
2\,5\,6\,3\,6\\
7\,4\,3\,6\,3\\
\hline
2\,6\,3\,4\,5\,4
\end{array}$$

The top line repeated with 2 subtracted from the right hand figure and 2 annexed on the left, completes the operation.

THIRD APPLICATION OF THE LIGHTNING METHOD:

$$\begin{array}{rl}
\left.\begin{array}{r}5\,6\,7\,8\\4\,3\,2\,1\end{array}\right\} & 9\text{'s}\\
\left.\begin{array}{r}6\,3\,9\,4\\3\,6\,0\,5\end{array}\right\} & 9\text{'s}\\
\left.\begin{array}{r}7\,8\,6\,3\\2\,1\,3\,6\end{array}\right\} & 9\text{'s}\\
\left.\begin{array}{r}4\,3\,8\,4\\5\,6\,1\,5\end{array}\right\} & 9\text{'s}\\
8\,9\,8\,7 & \text{Key Line.}\\
\hline
4\,8\,9\,8\,3 &
\end{array}$$

In this example we have four pairs of nines; therefore, subtract 4 from the right hand figure in the key line, and annex 4 on the left.

LEDGER COLUMNS.

A great part of the work of an accountant consists in adding long ledger columns, like the following. Let the pupil find the sum of the numbers in each, being as careful to obtain a correct result as he would be if he were to receive or pay the several amounts.

Combinations should be varied and made more difficult as proficiency is acquired. The secret in adding rapidly consists in familiarity with totals of combinations. Counting is not adding, and spelling is not reading.

8.37	.78	673.28
4.33	.47	597.84
7.62	.53	3426.87
.48	2.75	219.48
.97	1.20	8.37
2.50	4.37	167.84
6.19	8.29	5986.32
10.00	13.85	6749.31
4.28	2.00	4863.27
8.07	.62	7542.35
4.37	.25	2986.28
9.48	1.37	379.87
4.21	9.83	2.59
13.26	6.75	69.80
1.20	8.43	4060.75
.57	10.48	309.71
3.08	6.00	124.87
4.96	1.00	8520.06
.85	1.50	2493.28
4.00	7.69	48.75

THE EASY WAY TO ADD.

This method of adding numbers is especially designed for those whose minds are constantly employed with business affairs, the mind being relieved at intervals, and the mental labor of carrying over the sum of an entire column is obviated by the use of *Integers* or "catch figures."

EXAMPLE.

4^7
7
6
3^6
4
9
4^7
7
6
4^8
8
6
3^8
4
2
9
86

Process.—Begin at 9 to add as near 20 as you can, thus: $9 + 2 + 4 + 3 = 18$, reject the tens and place the 8 to the right of the 3, as in example; begin at 6 and add $6 + 8 + 4 = 18$, reject the tens, as before, and place 8 to the right of 4, as in example; begin at $6 + 7 + 4 = 17$, reject tens, place 7 to the right of 4, as in example; then $9 + 4 + 3 = 16$, reject tens, place 6 to the right of 3; then $6 + 7 + 4 = 17$, reject tens and place 7 to the right, as before; having arrived at the top of the column, add the figures in the new column, thus: $8 + 8 + 7 + 6 + 7 = 36$, or 3 tens and 6 units; place the 6 units as the unit's figure of the sum, having 3 tens to carry to 5 tens, the number of integers or catch figures already rejected. $3 + 5 = 8$ tens, which prefixed with the 6 makes 86 the sum.

N. B.—The small figures we set to the right are called *Integers*, "*tally*," or *catch figures*.

If upon arriving at the top of the column there should be one or more figures whose sum will not equal 10, add them to the sum of the figures in the new column; never place an extra figure in the new column unless it is an excess over 10.

$$\begin{array}{r} 3 \\ 2 \\ 4 \\ 3^{8} \\ 7 \\ 8 \\ 5^{5} \\ 4 \\ 6 \\ 6^{5} \\ 7 \\ 2 \\ \hline 57 \end{array}$$

EXAMPLE.—Proceed as before; begin at the bottom, $2 + 7 + 6 = 15$, reject the tens, place 5 to the right of 6; $6 + 4 + 5 = 15$, reject as before, place 5 to the right of 5; say $8 + 7 + 3 = 18$, reject as before, etc. Now we have three figures which do not add 10; add them to the new column and say $5 + 5 + 8 + 4 + 2 + 3 = 27$; place the 7 under the original column, add 2 to the number of tally figures, which is 3, thus: $3 + 2 = 5$, the tens figure in the sum, and makes 57, the answer.

Two or more columns can be added in the same manner.

$$\begin{array}{r} {}^{5}93^{9} \\ 65 \\ {}^{6}48 \\ 33 \\ 42^{8} \\ 53 \\ {}^{7}65 \\ 78 \\ 43^{2} \\ {}^{7}34 \\ 82 \\ 23 \\ \hline 659 \end{array}$$

EXAMPLE.—Proceed as in adding a single column. The sum of the first column being 49, we write the 9 and carry the 4 to the next column, thus: $4 + 2 + 8 + 3 = 17$, reject the tens as before, write the 7 to the left of the 3; then proceed as with a single column.

REMARK.—To add very long columns, it is better to add as near 30 as possible, instead of 20 as in above examples. The reason for suggesting this method is to decrease the number of "tally" figures. It must be remembered, however, by adopting 30 as a standard, that two tens will be rejected instead of one, as in the former examples, which will be observed in the following:

ADDITION.

3^{2}
2
4
8
5
3^{8}
1
6
7
4
2
5
3^{7}
7
8
4
5
1^{9}
2
9
3
4
2
3
5
6^{8}
4
4
5
4
3
2
2^{8}
4
3
6
4
5
4
162

EXAMPLE.

Process.—Here we begin at the bottom as before, adding over 20 and under 30, placing the excess or tally figure wherever it occurs in the line, thus: $4 + 5 + 4 + 6 + 3 + 4 + 2 = 28$, reject the 2 tens, place the 8 to the right of 2, etc.

By an inspection of the example it will be seen that $8 + 8 + 9 + 7 + 8 + 2 = 42$, the sum of the tally figures; place 2 as the unit's figure of the sum; carry 4 to twice the number of tally figures, since the twenties, not the tens, were rejected as tallies, so we say twice 6 are 12 and 4 to carry are 16; prefix this with the 2 units, and we have 162, the sum of the whole column.

NOTE.—The reason for adding numbers by tally figures must be clear, since it is nothing more than a condensing process, which can be briefly explained thus: take for instance the tally figures in the preceding example which are 8, 8, 9, 7, 8, 2, which when fully written out, would make the column read, But since the tens' figures are all alike, it is necessary to write the units only, and simply bear in mind that for every unit's figure written out we have two tens, and thus abbreviate.

28
28
29
27
28
22

REMARK.—This mode of adding is especially designed for those whose minds are constantly employed on business affairs, and who are apt to be interrupted. A little practice will enable anyone to add rapidly and *always* correctly without any mental labor or fatigue. But the young accountant, whose *business* it is to add, and *nothing else*, should rely entirely upon the mind and adopt the preceding rules.

Adding with Periods.

2·
4
5
4·
3
4
2·
4
3
2·
3
2
4
4
3
——
49

Another condensed method of Addition is by periods, which is illustrated in the following example:

Commence thus: 3 4 4 2 3 2 = 18, reject ten, place a period to the right of 2, carry 8 to the next figure, thus: 8 3 4 2 = 17, place the period to the right of 2, reject ten, carry 7 to next figure, 7 4 3 4 = 18; place period to the right of 4, reject ten, carry 8 to next figure, 8 5 4 2 = 19; reject ten, place 9 as the unit's figure in the sum; the number of periods, which are 4 in this case, will be the tens' figure in the sum of the column and completes the addition.

General Rules for Addition.

1. **Write numbers plainly and distinctly,** so that 9's may not be mistaken for 7's, or 5's for 3's.

2. **Write the numbers in vertical lines.** Irregularity in placing of figures is the cause of many errors.

3. **Think of results,** and not of the numbers themselves. Thus do not say 4 and 5 are 9, and 6 are 15, and 7 are 22, etc.; but 9, 15, 22, etc.

4. **Make combinations** of 10 or other numbers as often as possible, and add them as single numbers. Thus: in adding $\overline{9}$ $\overline{34}$ $\overline{73}$ $\overline{214}$ $\overline{9}$ $\overline{54}$ $\overline{82}$ $\overline{123}$, say 9 16 26 33 42 51 61 67, taking each group at a glance as a single number. When a figure is repeated several times, multiply instead of adding.

5. **In adding horizontally** begin at the left, since the eye is more accustomed to moving from left to right than from right to left.

6. **In adding long columns,** prove the work by adding each column separately in the opposite direction before adding the next column.

We believe that addition should be *drilled into* every boy, every day, from the beginning to the end of his school life. It is ten times more important than measures, multiples, interest, percentages, stocks, etc., all put together, and the sooner teachers thoroughly understand this fact the more practical and beneficial will be the result of our school system.

Pupils should be trained to add figures when placed in a horizontal position as well as when placed in a vertical position. In the following exercises add both ways, and then prove the results by adding them.

Ex. 1.	4621	3946	4256	8432	1562	=	22817
	421	5000	7060	85	984	=	****
	2012	1213	214	143	275	=	****
	1604	2103	1524	2138	4215	=	****
	2385	6214	3121	1562	1428	=	****
	11043	****	****	****	****	=	

Ex. 2.	$42	50	$13	40	$243	10	$3	04	$136	15		
	3	80	214	20	14	20	14	27	2	17		
		25	2	60	181	10	1	00	13	84		
	175	16	1	25	19	40	125	10	184	15		
	10	80	13	75	161	15	2	00	6	17		

MULTIPLICATION.

In the ordinary process of multiplication we obtain partial products and then add these together for the entire product. With a little wholesome practice, however, we multiply by a number consisting of several figures, without writing out the partial products. There are those who can multiply by a number consisting of 10 to 12 or even more digits, writing the result under the given number with great readiness. This is a very unusual degree of proficiency; but almost anyone can learn to do the same with a multiplier consisting of from 2 to 6 places. We indicate the method by the following problems and solutions. To make the operations easily understood, we have selected small numbers at first, and advance into higher and more difficult ones, step by step, and whoever studies the method thoroughly, and practices perseveringly, will be amply rewarded for the time devoted to the task.

EXAMPLES.—PROCESS.

$$\begin{array}{r} 23 \\ 32 \\ \hline 736 \end{array}$$

1st. $2\times3=6$, we write down the 6 for the units figure in the product.

2d. $(2\times2)+(3\times3)=13$, we write down the 3 for the ten's figure in the product, and reserve 1 to carry.

3d. $3\times2=6+1$ (we carried) $=7$, which completes the product.

EXAMPLE 2.

$$\begin{array}{r}78\\27\\\hline 2106\end{array}$$

1st. $7 \times 8=56$, we write down 6 as the unit's figure in the product, reserving 5 to carry.

2d. $(7 \times 7)+(2 \times 8)+5$ (to carry)$=70$; We write down the 0 for the ten's figure in the product, reserving 7 to carry.

3d. $2 \times 7+7=21$, which completes the product.

EXAMPLE 3.

$$\begin{array}{r}126\\35\\\hline 4410\end{array}$$

1st. $5 \times 6=30$, write down 0 as the unit's figure in the product, reserving 3 to carry.

2d. $(5 \times 2)+(3 \times 6)+3=31$, write 1 for the ten's figure in the product, reserving 3 to carry.

3d. $(5 \times 1)+(3 \times 2)+3=14$, write 4 for the hundred's place in the product, reserving 1 to carry.

4th. $3 \times 1+1=4$, which completes the product.

EXAMPLE 4.

$$\begin{array}{r}324\\436\\\hline 141264\end{array}$$

1st. $6 \times 4=24$, write 4 as the unit's figure in the product, reserve 2 to carry.

2d. $(3 \times 4)+(2 \times 6)+2=26$, write 6 as the ten's figure in the product, reserving 2 to carry.

3d. $(6 \times 3)+(4 \times 4)+(3 \times 2)+2=42$, write 2 as the hundred's figure in the product, reserving 4 to carry.

4th. $(3 \times 3)+(4 \times 2)+4=21$, write 1 as the thousand's figure in the product, reserving 2 to carry.

5th. $4 \times 3+2=14$, which completes the product.

THE SLIDING METHOD

—OF—

MULTIPLICATION.

Probably the easiest method to learn to multiply large numbers in a single line is the sliding method as used by Peter M. Deshong, which is in reality nothing more than cross multiplication, as illustrated; but for the beginner it is the best that can be adopted. When familiar with the slide the student can proceed without it, and perform operations astonishing to himself and those who witness the operation, the largest numbers being readily multiplied in a single line.

This method can easily be understood by following the examples and solutions here given with paper and pencil.

MULTIPLICATION.

EXAMPLE. — PROCESS.

```
   324
   436
 ------
141264
```

Write the multiplier on a slip of paper separate from that on which the multiplicand is written, in an inverted order, thus: 634; place this slip directly over the multiplicand, so that the 4 will be directly over the 6, thus:

```
  643
324
```

then say $6 \times 4 = 24$, write 4 as your unit's figure in the product, reserving 2 to carry; now slide the paper to the left so that 2 will come under 6, and 4 under 3, thus:

```
 634
324
```

now $(6 \times 2)+(4 \times 3)+2=26$, write 6 as the ten's figure, reserving 2 to carry; again slide the paper to the left so that 3 falls under 6, 2 under 3, and 4 under 4, thus:

```
634
324
```

and you have $(6 \times 3)+(3 \times 2)\times(4 \times 4)+2=42$, write 2 as hundred's figure in product, reserve 4 to carry; slide the paper again and the 3 will be under 3, and 2 under 4, thus:

```
634
 324
```

and you have $(3 \times 3)+(4 \times 2)+4=21$, write 1 as the thousand's figure in the product, reserving 2 to carry; now slide again, that 3 will be under 4, thus:

```
634
  324
```

and you have $3 \times 4+2=14$, which completes the product, 141264.

We have used the same figures in this, as in the preceeding example, and by close observation it can readily be seen that the work is all the same. The sliding method, however, saves the mental labor of

carrying over, in the mind, so many figures, which is quite wearisome to the unpractised mind. These additions will soon be performed at a glance, as the products are obvious without the formality of naming factors, which the student should never allow himself to do in any operation; it is just as easy to name products only. To understand these directions thoroughly, factors must be placed upon slips of paper, and the directions strictly complied with, which will give an insight into the mode of operation, and the reason will be better understood in ten minutes, than in three hours without them. When once familiar with the slide, the student may proceed without it. We will solve another example upon the same principle, naming products only, as it should be operated.

EXAMPLE. — PROCESS.

```
   5768
    324
-------
1868832
```

On a separate slip of paper, as before, invert the multiplier, thus: 423; place the multiplier so that 8 will be under 4, and you have 32; write 2 for units in product, reserve 3, carry 4; slide, say $24+16+3=43$, write 3, carry 4; slide, $28+12+24+4=68$, or thus: 28, 40, 64, 68, write 8, carry 6 as before, and slide again, 20, 34, 52, 58, write 8, carry 5; slide, 10, 31, 36, write 6, carry 3; slide once more, 15, 18: you have the complete product.

REMARK.—Proceed towards the left until the multiplicand passes from under the multiplier, each time adding what you carry to the several products that stand one over the other, and the result will be the product.

SLIDING METHOD.

EXAMPLE FIRST

5768 324 ——— 1,868,832	Reverse the multiplier, thus: **423.**

Reading products only.

423 5768	32
423 5768	24 + 16 + 3 = 43
423 5768	28 + 12 + 24 + 4 = 68
423 5768	20 + 14 + 18 + 6 = 58
423 5768	10 + 21 + 5 = 36
423 5768	15 + 3 = 18

EXAMPLE SECOND.

4658 623 ——— 2,901,934	Reverse the multiplier, thus: **326.**

326 4658	24
326 4658	16 + 15 + 2 = 33
326 4658	48 + 10 + 18 + 3 = 79
326 4658	30 + 12 + 12 + 7 = 61
326 4658	36 + 8 + 6 = 50
326 4658	24 + 5 = 29

CONTRACTIONS

IN

MULTIPLICATION.

Contractions can often be advantageously employed in business calculations; but, like by-paths in a forest, they are convenient only to those who know the whole ground. Strangers will do better to keep the highway.

TO MULTIPLY BY ELEVEN.

RULE.—Add the figures in the multiplicand, after the first, from right to left.

APPLICATION 1.—45 X 11. $4 + 5 = 9$; place this sum between 4 and 5, thus 495.

35 X 11. $3 + 5 = 8$; place this sum between 3 and 5, thus 385.

42 X 11 = 462.	63 X 11 = 693.
81 X 11 = 891.	44 X 11 = 484.
95 X 11 = 1045.	49 X 11 = 539.

APPLICATION 2.—345 X 11. Here we write 5; we say $4 + 5 = 9$ write 9; then $4 + 3 = 7$; write 7; then write 3, thus 3795.

254 X 11. Write 4 for the first figure in the product; $4 + 5 = 9$, write 9 for the second figure; $5 + 2 = 7$ which is the third figure, and write 2 for the last figure, and we have 2794.

325 X 11 = 3575.	127 X 11 = 1397.
353 X 11 = 3883.	272 X 11 = 2992.

NOTE.—If the sum of two figures is over 9, carry the one to the next figure.

APPLICATION 3.—58 X 11 = 638; here we write 8 as the first figure, and say $5 + 8 = 13$; write 3 for the middle figure, and carry the one to the next figure 5, making the product 638.

75 X 11 = 825.	95 X 11 = 1045.
385 X 11 = 4235.	5863 X 11 = 64493.

TO MULTIPLY TWO FIGURES BY TWO FIGURES WHEN THE TENS ARE ALIKE.

To multiply 87 by 82.

Multiply units by units for the first figure of the product, the sum of the units by tens for the second figure, and tens by tens for the remaining figures, carrying when necessary.

87	$7 \times 2 = 14$, carry 1.
82	$8 \times (7 + 2) = 8 \times 9 = 72$, and $1 = 73$.
7134	$8 \times 8 = 64$, and 7 to carry $= 71$.

Exercises.

81 × 87	81 × 87	116 × 117
62 × 63	47 × 44	127 × 122
54 × 55	56 × 52	107 × 105
43 × 41	79 × 75	125 × 122
27 × 22	44 × 43	113 × 114

CONTRACTIONS.

TO SQUARE ANY NUMBER OF NINES.

Rule.—Write from left to right as many nines, less one, as the given number contains, an 8, as many ciphers as nines, and 1.

Thus the square of	999	998001
	9999	99980001
	99999	9999800001

TO MULTIPLY BY ANY NUMBER OF NINES.

Rule.—Annex as many ciphers to the right of the multiplicand as there are nines in the multiplier, and from this number subtract the multiplicand; the remainder will be the product required.

EXAMPLE.—37645 × 9999.

$$\begin{array}{r} 376450000 \\ 37645 \\ \hline 376412355 \end{array}$$

The reason is obvious. By annexing four ciphers, we multiply the given number 10000 times; and by subtracting the given number, we have the product one less than 10000, or 9999 times the number.

TO MULTIPLY BY ANY NUMBER ENDING IN NINE.

Rule.—Multiply by the next higher number, and subtract the multiplicand.

EXAMPLE.—42 × 39.

$39 + 1 = 40$
$42 \times 40 = 1680$ minus $42 = 1638$.—*Ans.*

CONTRACTIONS.

TO MULTIPLY BY ANY NUMBER FROM TWELVE TO TWENTY.

Rule. — Multiply in regular succession the figures of the multiplicand by the unit's figure of the multiplier, and add to the product of each multiplication that figure in the multiplicand which stands next on the right of the one which you multiply; add, also, the figure to carry, if any.

EXAMPLE. —
$$\begin{array}{r} 36435 \\ 13 \\ \hline 473655 \end{array}$$

Here we say $3 \times 5 = 15$; write down 5, carry 1; say $3 \times 3 + 1 + 5$, the figure which stands on the right of $3 = 15$; write 5, carry 1; say $3 \times 4 + 1 + 3 = 16$; write 6 and carry 1; $3 \times 6 + 1 + 4 = 23$; write 3, carry 2; $3 \times 3 + 2 + 6 = 17$; write 7, carry 1 to $3 = 4$.

TO MULTIPLY BY 21, 31, 41, 51, 61, 71, 81, 91.

Rule. — Write down the units figure of the multiplicand as the first figure of the product. Multiply in regular succession every figure in the multiplicand by the left hand figure of the multiplier, and to each product add the figure which stands next on the left of that which you multiply, and you have the required product.

EXAMPLE:

$$\begin{array}{r} 3725 \\ 21 \\ \hline 78225 \end{array}$$

Here we write the 5 as the units figure in the multiplier; then say $5 \times 2 + 2 = 12$; write 2, carry 1; say $2 \times 2 + 1 + 7 = 12$; write 2, carry 1; say $2 \times 7 + 1 + 3 = 18$: write 8, carry 1; then say $\cdot \times 3 + 1 = 7$, which completes the product.

CONTRACTIONS.

TO MULTIPLY ANY NUMBERS OF TWO PLACES EACH, WHEN THE UNITS OR TENS ARE ALIKE.

Rule. — Multiply units by units; then, if the units are alike, multiply the sum of the tens, and the tens by the tens. If the tens are alike, multiply the sum of the units by the tens, and the tens by tens; in all cases carrying as usual.

EXAMPLE 1:

34	$4 \times 4 = 16$; write 6, carry 1.
54	$5 + 3 \times 4 + 1 = 33$; write 3, carry 3.
——	$5 \times 3 + 3 = 18$, which completes the product.
1836	

EXAMPLE 2:

45	$5 \times 3 = 15$; write 5, carry 1.
43	$5 + 3 \times 4 + 1 = 33$; write 3, carry 3.
——	$4 \times 4 + 3 = 19$, completes the product.
1938	

This rule will apply to the square of any number. It is the most useful of all contractions, and should be carefully studied.

TO MULTIPLY BY NUMBERS WHICH ARE FROM ONE TO TWELVE LESS THAN ONE HUNDRED, ONE THOUSAND, ETC.

Rule. — Multiply the multiplicand by the difference between the multiplier and 100, 1000, &c., and subtract the product from the product of the multiplicand by 100, 1000, &c.

EXAMPLE. — Multiply 35 by 98.

$$98 — 100 — 2.$$
$$35 \times 2 = 70$$
$$35 \times 100 = 3500.$$
$$3500 - 70 = 3430. \text{ — } \textit{Ans.}$$

When from 1 to 12 more than 100, add the product of the multiplicand by the unit figure, after annexing the required number of ciphers, thus:

EXAMPLE. — Multiply 325 by 102.

$$325 \times 100 = 32500.$$
$$325 \times 2 = 650 + 32500 = 33150.$$

CONTRACTIONS.

WHEN THE SUM OF THE UNITS IS TEN, AND THE TENS ARE ALIKE.

Method. — Multiply the units and write the result as the first two figures in the product. Then call the tens figure one more and write their product for the last two figures in the final product.

EXAMPLE.

$$\begin{array}{r} 86 \\ 84 \\ \hline 7224 \end{array} \qquad \begin{array}{r} 4 \times 6 = 24. \\ 8 + 1 \times 8 = 72. \end{array}$$

93	85	48	63	56	71	66
97	85	42	67	54	79	64

This contraction will only apply where the units equal ten and the tens are alike, as in above example. If the product of units does contain ten, as in 9×10, the place of tens must be supplied with a cipher.

TO SQUARE ANY NUMBER ENDING IN FIVE.

Method. — Multiply the figure preceding the units as they will stand by the next higher number, and annex 25 to the product.

EXAMPLE.

$$\begin{array}{r} 75 \\ 75 \\ \hline 5625 \end{array} \qquad \begin{array}{c} 7 + 1 \times 7 = 56. \\ \text{Annex } 25. \end{array}$$

WHAT IS THE SQUARE OF

25? 35? 45? 75? 85? 95? 105? 115? 125?
135? 145? 155? 165? 175? 185? 195? 205?

CONTRACTIONS.

TO FIND THE PRODUCT OF ANY TWO NUMBERS WHOSE UNIT FIGURES ARE FIVE.

Method. — Take the product of the figures preceding the 5 in each number, increase this by one-half of the sum of these figures, and prefix the result to 25.

EXAMPLE.

$$\begin{array}{r} 25 \\ 45 \\ \hline 1125 \end{array} \qquad 4 \times 2 + 3 = 11.$$

WHAT IS THE VALUE OF

25×45? 55×75? 75×95? 65×95?
35×85? 85×45? 155×35? 165×45?
185×65? 175×65? 225×105?

NOTE. — If the sum of the figures preceding the 5 is odd, when we take one-half of it, the one-half or five-tenths which remains must be added to the figure 2 of the 25; or we may take one-half of the next smaller number, and use 75 as the suffix.

TO MULTIPLY BY TWO FIGURES AT ONCE.

Rule. — Multiply both figures in the multiplier by each figure in the multiplicand separately.

NOTE. — When large numbers are to be multiplied, for the purpose of remembering which figure has been used, place a dot over each figure of the multiplicand as soon as multiplied.

EXAMPLE.

$$\begin{array}{r} 3265 \\ 24 \\ \hline 78360 \end{array} \qquad \begin{array}{l} 5 \times 24 = 120 \\ 6 \times 24 + 12 = 156 \\ 2 \times 24 + 15 = 63 \\ 3 \times 24 + 6 = 78 \end{array}$$

To Multiply by Aliquot Parts of 100, 1000, Etc.

It is very important for an accountant to have a perfect knowledge of the table of Aliquot Parts of 100 and 1000. All goods sold at wholesale are bought and sold by these calculations, and those not familiar with the operation will often lose much valuable time in obtaining a correct result. By this method they can arrive at the result in one-tenth of the time, and are not so apt to make mistakes.

ALIQUOT PARTS.

Of 10.

$2\frac{1}{2} = \frac{1}{4}$	$1\frac{2}{3} = \frac{1}{6}$	$1\frac{1}{4} = \frac{1}{8}$
$3\frac{1}{3} = \frac{1}{3}$	$1\frac{3}{7} = \frac{1}{7}$	$1\frac{1}{9} = \frac{1}{9}$

Of 100.

$6\frac{1}{4} = \frac{1}{16}$	$16\frac{2}{3} = \frac{1}{6}$	$50 = \frac{1}{2}$
$8\frac{1}{3} = \frac{1}{12}$	$25 = \frac{1}{4}$	$62\frac{1}{2} = \frac{5}{8}$ or 1-16 of 1000
$12\frac{1}{2} = \frac{1}{8}$	$33\frac{1}{3} = \frac{1}{3}$	$75 = \frac{3}{4}$
$14\frac{2}{7} = \frac{1}{7}$	$37\frac{1}{2} = \frac{3}{8}$	$87\frac{1}{2} = \frac{7}{8}$
	$18\frac{3}{4} = \frac{3}{16}$	$31\frac{1}{4} = \frac{5}{16}$

Of 1000.

$83\frac{1}{3} = \frac{1}{12}$	$166\frac{2}{3} = \frac{1}{6}$	$333\frac{1}{3} = \frac{1}{3}$
$125 = \frac{1}{8}$	$250 = \frac{1}{4}$	$375 = \frac{3}{8}$

$625 = \frac{5}{8}$ or 1-16 of 10,000.

$833\frac{1}{3} = \frac{5}{6}$ or 1-16 of 1,000. $875 = \frac{7}{8}$.

EXAMPLE 1.—Multiply 464 by 25 = 11600.

$$4\,)\,\underline{46400}$$
$$11600$$

This is, in effect, the same as to multiply by 100 we divide by 4 because 25 is $\frac{1}{4}$ of 100, which is the same as multiplying by 25. In the same manner, annex two ciphers and divide by 2 multiplies by 50; annex two ciphers and divide by 8 multiplies by $12\frac{1}{2}$; or annex two ciphers and divide by 8 to multiply by 125, etc., etc.

This same principle may be applied in any Aliquot Part of 10, 100, 1000, as shown in preceding table.

RULE.—Add ciphers to the multiplicand and divide by the number, as the multiplier is a part of 100 or 1000. When the multiplicand is a mixed number, reduce the fraction to a decimal and proceed as before.

EXAMPLE 1.—Multiply 434 by $2\frac{1}{2}$.

$$4\,)\,\underline{43400}$$
$$10850$$

EXAMPLE 2.—Multiply 535 by 25.

$$4\,)\,\underline{53500}$$
$$13375$$

EXAMPLE 3.—Multiply 5642 by $3\frac{1}{3}$.

$$3\,)\,\underline{56420}$$
$$18806\tfrac{2}{3}$$

EXAMPLE 4.—Multiply 4321 by $33\frac{1}{3}$.

$$3\,)\,\underline{432100}$$
$$144033\tfrac{2}{3}$$

EXAMPLE 5.—Multiply 1234 by $1\frac{1}{3}$.

$$6\,)\,\underline{12340}$$
$$2056\tfrac{2}{3}$$

EXAMPLE 6.—Multiply 2245 by $1\frac{3}{7}$.

$$7\,)\,\underline{22450}$$
$$3207\tfrac{1}{7}$$

EXAMPLE 7.—Multiply 4456 by $1\frac{1}{4}$.

$$8\,)\,\underline{44560}$$
$$5570$$

EXAMPLE 8.—Multiply 5644 by $1\frac{1}{9}$.

$$9\,)\,\underline{56440}$$
$$6271\tfrac{1}{9}$$

EXAMPLE 9.—Multiply 4324 by $6\frac{1}{4}$.

$$16\,)\,\underline{432400}$$
$$27025$$

EXAMPLE 10.—Multiply 5642 by $8\frac{1}{3}$.

$$12\,)\,\underline{564200}$$
$$47016\tfrac{2}{3}$$

EXAMPLE 11.—Multiply 5648 by $12\frac{1}{2}$.

$$8\,)\,\underline{564800}$$
$$70600$$

EXAMPLE 12.—Multiply 6843 by $14\frac{2}{7}$.

$$7\,)\,\underline{684300}$$
$$97757\tfrac{1}{7}$$

EXAMPLE 13.—Multiply 7824 by $16\frac{2}{3}$.

$$6\,)\,\underline{782400}$$
$$130400$$

EXAMPLE 14.—Multiply 7846 by $83\frac{1}{3}$.

$$12\,)\,\underline{7846000}$$
$$653833\tfrac{1}{3}$$

EXAMPLE 15.—Multiply 7896 by 125

$\frac{1}{8}$ of 1000. $\quad 8\,)\,7896000$

$\quad\quad\quad\quad\quad\quad 987000$

EXAMPLE 16.—Multiply 1246 by $166\frac{2}{3}$.

$\frac{1}{6}$ of 1000. $\quad 6\,)\,1246000$

$\quad\quad\quad\quad\quad\quad 207666\frac{2}{3}$

EXAMPLE 17.—Multiply 8453 by 250.

$\frac{1}{4}$ of 100. $\quad 4\,)\,8453000$

$\quad\quad\quad\quad\quad\quad 2113250$

EXAMPLE 18.—Multiply 4642 by 625.

$\frac{1}{16}$ of 10000. $\quad 16\,)\,46420000$

or, $\quad\quad\quad\quad\quad\quad 2901250$

$\frac{5}{8}$ of 1000. $\quad 8\,)\,4642000$

$\quad\quad\quad\quad\quad\quad 580250$

$\quad\quad\quad\quad\quad\quad\quad\quad 5$

$\quad\quad\quad\quad\quad\quad 2901250$

EXAMPLE 19.—Multiply 5642 by $833\frac{1}{3}$.

$\frac{1}{12}$ of 10000. $\quad 12\,)\,56420000$

or, $\quad\quad\quad\quad\quad\quad 4701666\frac{2}{3}$

$\frac{5}{6}$ of 1000. $\quad 6\,)\,5642000$

$\quad\quad\quad\quad\quad\quad 940333\frac{1}{3}$

$\quad\quad\quad\quad\quad\quad\quad\quad 5$

$\quad\quad\quad\quad\quad\quad 4701666\frac{2}{3}$

EXAMPLE 20.—Multiply 1342 by 875.

$\frac{7}{8}$ of 1000. $\quad 8\,)\,1342000$

$\quad\quad\quad\quad\quad\quad 167750$

$\quad\quad\quad\quad\quad\quad\quad\quad 7$

$\quad\quad\quad\quad\quad\quad 1174250$

APPLICATION OF THE TABLE OF ALIQUOT PARTS.

In order to give an idea of rapid calculation in multiplication, a few examples will here be given, and from these others may be created without limit by anyone:

35 yards cloth @ \$2.50. Add one cipher and divide by 4. Answer, $87\frac{1}{2}$ or \$87.50.

216 yards cloth @ \$2.25. Multiply $\$2\frac{1}{4}$ by setting down the amounts thus:

$$\begin{array}{r} 432 \\ 54 \\ \hline \$486 \end{array}$$

48 yards cloth @ $\$2.12\frac{1}{2}$. Multiply by $\$2\frac{1}{8}$ in same manner as by $\$2\frac{1}{4}$.

55 yards cloth @ \$1.95. Move decimal point in price two places to the right, divide 195 by 2 and add $\frac{1}{10}$ thus:

$$\begin{array}{r} 97.50 \\ 9.75 \\ \hline \$107.25 \end{array}$$

162 yards cloth @ \$1.80. Multiply by 2 and deduct $\frac{1}{10}$ thus:

$$\begin{array}{r} 324 \\ 32.40 \\ \hline \$291.60 \end{array}$$

36 yards cloth @ \$1.75. Multiply by 2 and deduct $\frac{1}{8}$, thus:

$$\begin{array}{r} 72 \\ 9 \\ \hline \$63 \end{array}$$

29 yards cloth @ $\$1.62\frac{1}{2}$. Add $\frac{1}{2}$ and $\frac{1}{8}$ to the whole, thus:

$$\begin{array}{r} 29 \\ 14.50 \\ 3.62\frac{1}{2} \\ \hline \$47.12 \end{array}$$

$114\frac{1}{2}$ yards cloth @ \$1.50. Add $\frac{1}{2}$, thus:

$$\begin{array}{r} 114.50 \\ 57.25 \\ \hline \$171.75 \end{array}$$

$37\frac{3}{4}$ yards cloth @ \$1.25. Add $\frac{1}{4}$, thus:

$$\begin{array}{r} 37.75 \\ 9.44 \\ \hline \$47.19 \end{array}$$

83 yards cloth @ \$1.20. Add $\frac{1}{5}$, thus:

$$\begin{array}{r} 83 \\ 16.60 \\ \hline \$99.60 \end{array}$$

APPLICATION OF THE TABLE OF ALIQUOT PARTS.

50 yards cloth @ $1.18. Find $\frac{1}{2}$ of $118 = $59.

75 yards cloth @ $1.14. Deduct from $114 $\frac{1}{4}$ of that amount, thus:	114 28.50 $85.50
24 yards cloth @ 95c. Deduct $\frac{1}{20}$ from $24, thus:	24 1.20 $22.80
68$\frac{1}{2}$ yards cloth @ 75c. Deduct $\frac{1}{4}$, thus:	68.50 17.12 $51.38
46 yards cloth @ 55c. Find $\frac{1}{2}$ and add to same $\frac{1}{10}$, thus:	23 2.30 $25.30
32 yards cloth @ 45c. Find $\frac{1}{2}$ and deduct $\frac{1}{10}$, thus:	16 1.60 $14.40

96 yards cloth @ 25c. Find $\frac{1}{4}$ of $96 = $24.

The reason for making the computations in this manner will at once be apparent, from the fact that when the price is either more or less than $1, the fractional part of a dollar is either taken from, or added to, the amount that sum would be if @ $1 per yard. It is sometimes more convenient to call the number of yards the price, and the price the number of yards, in order to make the computation, as in the example above; 50 yards @ $1.18 would be the same as 118 yards @ 50c.; or to say, if 100 yards @ $1.18 would be $118, 50 yards would be half of that amount, or $59. In the first example we say, if 35 yards @ $10 per yard would be $350, at $2.50 per yard it would be one-fourth of that amount, or $87.50.

It is of course expected that the student will perform these operations in his mind. Nothing is more desirable for an Entry Clerk or Book-keeper than to have a thorough knowledge of the aliquot parts of 100 or 1000. Many remunerative situations have been obtained by those who thoroughly understood the practice, though their general education was very limited. Constant practice will enable anyone to give the products as fast as the questions are given.

Another mode of multiplying, when the multiplier can be divided into factors, is an improvement on the common method ; but to multiply in a single line is still better. The objection to this method is that when an error occurs in the first line it will run into the second.

EXAMPLE :—

$$\begin{array}{r} 1234 \\ 124 \\ \hline 4936 \\ 14808 \\ \hline 153016 \end{array}$$

Here we multiply through by 4 ; now, since **12 is** 3 times 4, if we multiply the first line by 3 we have the product of 12 in a single line. Quite a variety of examples can be worked in this way.

HOW TO PROVE MULTIPLICATION BY CASTING OUT THE NINES.

RULE.—Find out the excess over nine in your multiplicand and multiplier, and if the excess in the product of these excesses is the same as the excess in the product, the operation is correct.

EXAMPLE :—

$$\begin{array}{rcc} 1234 & 1 & \\ 124 & 7 & \\ \hline 153016 & 7 & \end{array} \qquad \begin{array}{ccc} & 7 & \\ 7 & \times & 1 \\ & 7 & \end{array}$$

MULTIPLICATION

—BY—

SQUARING NUMBERS.

$1^2 = 1$	$12 = 144$
$2^2 = 4$	$13^2 = 169$
$3^2 = 9$	$14^2 = 196$
$4^2 = 16$	$15^2 = 225$
$5^2 = 25$	$16^2 = 256$
$6^2 = 36$	$17^2 = 289$
$7^2 = 49$	$18^2 = 324$
$8^2 = 64$	$19^2 = 361$
$9^2 = 81$	$20^2 = 400$
$10^2 = 100$	$21^2 = 441$
$11^2 = 121$	$22^2 = 484$

NOTE.—The product of any two numbers is equal to the square of the mean, minus the square of half their difference. The *mean* is a number as much greater than the less, as it is less than the greater.

RULE.—From the square of the mean subtract the square of the difference between either of the given numbers and the mean.

EXAMPLE 1.

$$\begin{array}{r} 17 \\ 13 \\ \hline 221 \end{array} \qquad 15^2 - 4 = 221$$

REMARK.—15 is as much greater than 13 as it is less than 17, 15 is therefore the mean between 17 and 13. Half the difference is 2, the square of 2 is 4; or, we may say, the difference between the mean and either of the given numbers is 2, square of which is 4.

EXAMPLE 2.---What cost 19 books at 13 cts. each?

SOLUTION.—The mean of 13 and 19 is 16, the square of which is 256 — 9, the square of *half* the difference of the given numbers = 245.

EXAMPLE 3.—What cost 13 tons of hay at $14.00 a ton?

Thus: $13^2 + 13 = 182$

Where the difference of two numbers is a unit, we add the less number to its square, for

$$13 \times 13 = 14 \text{ times } 13.$$

EXAMPLE 4.—What cost 16 ounces of gold dust at $17.00 an ounce?

Thus: $16^2 + 16 = 272.$

Table of Square Numbers continued.

	$26^2 = 676$
	$27^2 = 729$
$23^2 = 529$	$28^2 = 784$
$24^2 = 576$	$29^2 = 841$
$25^2 = 625$	$30^2 = 900$

Here let the student observe that the two right-hand figures in the square of any number, as much less than 25 as another is greater, are the same in one case as in the other. For example, in the squares of 23 and 27, the one as much less as the other is greater than 25, the two right-hand figures are the same. This law holds true in all cases.

EXAMPLE 1.—What cost 23 shad at 27 cts. each?

$$25^2 - 4 = 621.$$

EXAMPLE 2.—What cost 26 tons of hay at $27.00 a ton?

$$26^2 + 26 = 703$$

Table of Square Numbers continued.

$31^2 = 961$	$19^2 = 361$
$32^2 = 1024$	$18^2 = 324$
$33^2 = 1089$	$17^2 = 289$
$34^2 = 1156$	$16^2 = 256$
$35^2 = 1225$	$15^2 = 225$
$36^2 = 1296$	$14^2 = 196$
$37^2 = 1369$	$13^2 = 169$
$38^2 = 1444$	$12^2 = 144$
$39^2 = 1521$	$11^2 = 121$
$40^2 = 1600$	$10^2 = 100$

The first column is placed here to afford the student an opportunity of observing that the *two right-hand figures* in the square of any number which is as much less than 25 as the other is greater, are the same in the former case as in the latter; or, which is the same thing, the two right-hand figures in the square of any number as much *less* than 20 as another is *greater* than 30 are the same in one case as in the other.

Table of Squares continued.

$41^2 = 1681,\ 9^2$	$46^2 = 2116,\ 4^2$
$42^2 = 1764,\ 8^2$	$47^2 = 2209,\ 3^2$
$43^2 = 1849,\ 7^2$	$48^2 = 2304,\ 2^2$
$44^2 = 1936,\ 6^2$	$49^2 = 2401,\ 1^2$
$45^2 = 2025,\ 5^2$	$50^2 = 2500,\ 0^2$

The student need have no difficulty in remembering the squares in the above columns. Observe that the two right-hand figures in the square are in every instance the square of the difference between the units figure of the root and 10; the two left-hand figures in the square will also be easily remembered if we observe that the number is formed by adding 1 less than the right-hand or units figure of the root

to the square of the left-hand, or tens figure; thus, in the square of 47 the left-hand figures are 22, or 1 less than 7 added to the square of 4, thus:

$$4^2 = 16 + (7 - 1) = 22.$$

The right-hand fignres are obtained by subtracting 7 from 10 $= 3$; the square of $3 = 9$

NOTE.—When the square of the difference between right-hand figures and ten is not over 9, prefix a cipher, as in above case.

EXAMPLE 2.—$43^2 = 10 - 3 = 7^2 = 49$, right-hand figures; $4 \times 4 + 2 = 18$, left-hand figures; therefore $43^2 = 1849$.

Table of Squares continued.

$51^2 = 2601$	$56^2 = 3136$
$52^2 = 2704$	$57^2 = 3249$
$53^2 = 2809$	$58^2 = 3364$
$54^2 = 2916$	$59^2 = 3481$
$55^2 = 3025$	$60^2 = 3600$

This square can also easily be remembered if we observe that the left-hand figures in every instance may be produced by adding the right-hand figure of the root to the square of the left-hand figure, and the number expressed by the two right-hana figures in the square, is the square of the right-hand figures in the root.

Example, square 56. Here we say $6^2 = 36$, which are the right-hand figures in the square. Again, $5^2 + 6 = 31$, which are the left hand figures in the square.

Table of Squares continued.

$61^2 = 3721$	$11^2 = 121$	$70^2 = 4900$	$24^2 = 576$
$62^2 = 3844$	$12^2 = 144$	$71^2 = 5041$	$25^2 = 625$
$63^2 = 3969$	$13^2 = 169$	$72^2 = 5184$	$26^2 = 676$
$64^2 = 4096$	$14^2 = 196$	$73^2 = 5329$	$27^2 = 729$
$65^2 = 4225$	$15^2 = 225$	$74^2 = 5476$	$28^2 = 784$

$66 = 4356$	$16^2 = 256$	$75^2 = 5625$	$29^2 = 841$
$67^2 = 4489$	$17^2 = 289$	$76^2 = 5776$	$30^2 = 900$
$68^2 = 4624$	$18^2 = 324$	$77^2 = 5929$	$21^2 = 441$
$69^2 = 4761$	$19^2 = 361$	$78^2 = 6084$	$22^2 = 484$
$70^2 = 4900$	$20^2 = 400$	$79^2 = 6241$	$23^2 = 529$
		$80^2 = 6400$	

In the above tables the columns of squares on the right are thus placed with their roots, in order that the student may associate them with the squares of the left-hand column. He will of course observe that two figures (units and tens) are in regular succession, the same in one column as in the other. In the following columns the same principle will be observed.

$81^2 = 6561$	$19^2 = 361$	$91^2 = 8281$	$9^2 = 81$
$82^2 = 6724$	$18^2 = 324$	$92^2 = 8464$	$8^2 = 64$
$83^2 = 6889$	$17^2 = 289$	$93^2 = 8649$	$7^2 = 49$
$84^2 = 7056$	$16^2 = 256$	$94^2 = 8836$	$6^2 = 36$
$85^2 = 7225$	$15^2 = 225$	$95^2 = 9025$	$5^2 = 25$
$86^2 = 7396$	$14^2 = 196$	$96^2 = 9216$	$4^2 = 16$
$87^2 = 7569$	$13^2 = 169$	$97^2 = 9409$	$3^2 = 9$
$88^2 = 7744$	$12^2 = 144$	$98^2 = 9604$	$2^2 = 4$
$89^2 = 7921$	$11^2 = 121$	$99^2 = 9801$	$1^2 = 1$
$90^2 = 8100$	$10^2 = 100$	$100^2 = 10000$	$0^2 = 0$

The square of last column may be obtained by adding twice the right-hand figure of the root to 80, thus: $91^2 = 2 \times 1 = 2 + 80 = 82$.

Example 2.—$99^2 = 80 + 18 = 98$, left-hand figures in the square.

TO SQUARE ANY NUMBER ENDING IN 5.

Rule.—Multiply the part preceding the units by itself, increase by a unit and prefix the product to 25, thus: $65^2 = 6 \times 7 = 42$; prefix $25 = 4225$.

Example 2.—$72^2 = 7 \times 8 = 56$; prefix $25 = 5625$.

DIVISION.

The work in division can be abbreviated by not writing out the product figures, and finding the remainders as we pass along.

RULE. — Subtract each product figure as it is formed (that is, the right-hand figure of the product), and when it is greater than the figure from which you subtract, carry one more to the next product figure than you would otherwise carry.

EXAMPLE: —

```
29 ) 15341 ( 529
        8
       26
```

Say $5 \times 9 = 45$. 5 from $13 = 8$; then $5 \times 2 + 4 + 1 = 15$. Our next quotient figure is 2; now say $2 \times 9 = 18$; 8 from $14 = 6$; $2 \times 2 + 1 + 1 = 6$, subtracted from 8 leaves 2, or 26, our next, etc.

The Italian method of dividing is to place the divisor to the right of the dividend, and the quotient underneath it. The advantage in this is, you bring the operation closer together and can readily multiply the quotient by the divisor, to prove the work.

EXAMPLE: —

```
15341 (  29
   8    529
  26
```

CONTRACTIONS IN DIVISION.

The table of Aliquot Parts on page 25 can be applied to division.

To divide by 25 multiply the dividend by 4 and point off two figures to the right-hand of the product as so many hundredths, or take one-fourth of the two right-hand figures of the product as so many twenty-fifths.

EXAMPLE 1.— Divide 3757 by 25.

$$\begin{array}{r} 3757 \\ 4 \\ \hline 15028 \end{array} \quad \text{or,}\ 150\tfrac{7}{25}$$

EXAMPLE 2.— Divide 437924 by 125.

$$\begin{array}{r} 437924 \\ 8 \\ \hline 3503392 \end{array} = 3503\tfrac{49}{125}$$

This method will hold good through the entire table of Aliquot Parts. It is simply the reverse of multiplication. Another convenient mode of dividing is to reduce the divisor into factors.

EXAMPLE.— Divide 34569 by 24.

$24 = 6 \times 4$

$$\begin{array}{rl} 6\,)\,34569 & \\ \hline 4\,)\,5761 & \text{— 3 over.} \\ \hline 1440 & \text{— 1 over.} \end{array}$$

$6 \times 1 + 3 = 9$ true remainder.

$$\text{or,}\ \begin{array}{rl} 4\,)\,34569 & \\ \hline 6\,)\,8642 & \text{— 1} \\ \hline 1440 & \text{— 2} \end{array}$$

$2 \times 4 + 1 = 9$ true remainder.

FRACTIONS.

A *Proper Fraction* is one whose value is less than a unit, as $\frac{2}{3}$, $\frac{3}{4}$.

An *Improper Fraction* is one whose value is equal to or more than a unit, as $\frac{2}{2}$, $\frac{5}{3}$.

A *Mixed Fraction* consists of a whole number and a fraction, as $2\frac{1}{4}$, $4\frac{1}{2}$.

1.—Multiplying the numerator of a fraction by any number multiplies the value of the fraction by that number.

2.—Dividing the numerator of a fraction by any number divides the value of the fraction by that number.

3.—Multiplying the denominator of a fraction by any number divides the value of the fraction by that number.

4.—Dividing the denominator of a fraction by any number multiplies the value of the fraction by that number.

5.—Multiplying both numerator and denominator of a fraction by any number does not change the value of the fraction.

6.—Dividing both numerator and denominator of a fraction by any number does not change the value of the fraction.

To Higher Terms

How many fourths in $\frac{1}{2}$?

Solution. — In 1 there are $\frac{4}{4}$, and in $\frac{1}{2}$ there are $\frac{1}{2}$ of $\frac{4}{4}$, or $\frac{2}{4}$.

How many sixths in	$\frac{1}{2}$?	$\frac{1}{3}$?	$\frac{2}{3}$?	$\frac{3}{2}$?
How many eighths in	$\frac{1}{2}$?	$\frac{1}{4}$?	$\frac{2}{4}$?	$\frac{3}{4}$?
How many tenths in	$\frac{1}{2}$?	$\frac{1}{5}$?	$\frac{3}{5}$?	$\frac{4}{5}$?
How many twelfths in	$\frac{1}{3}$?	$\frac{1}{4}$?	$\frac{1}{6}$?	$\frac{4}{6}$?
How many fourteenths in	$\frac{1}{2}$?	$\frac{2}{7}$?	$\frac{4}{7}$?	$\frac{5}{7}$?
How many fifteenths in	$\frac{2}{3}$?	$\frac{3}{5}$?	$\frac{4}{5}$?	$\frac{5}{3}$?
How many sixteenths in	$\frac{1}{4}$?	$\frac{3}{4}$?	$\frac{2}{8}$?	$\frac{5}{8}$?
How many eighteenths in	$\frac{2}{3}$?	$\frac{4}{6}$?	$\frac{3}{9}$?	$\frac{8}{9}$?
How many twentieths in	$\frac{4}{5}$?	$\frac{3}{4}$?	$\frac{7}{10}$?	$\frac{3}{5}$?

To Lower Terms.

How many thirds are equal to $\frac{4}{6}$?

Solution.—$\frac{1}{3}$ is equal to $\frac{2}{6}$, therefore $\frac{1}{2}$ of the number of sixths equals the number of thirds; $\frac{1}{2}$ of 4 is 2.

How many halves in	$\frac{2}{4}$?	$\frac{6}{4}$?	$\frac{8}{4}$?	$\frac{10}{4}$?
How many thirds in	$\frac{2}{6}$?	$\frac{4}{6}$?	$\frac{6}{9}$?	$\frac{8}{12}$?
How many fourths in	$\frac{1}{2}$?	$\frac{6}{8}$?	$\frac{9}{12}$?	$\frac{10}{8}$?
How many sixths in	$\frac{10}{12}$?	$\frac{9}{18}$?	$\frac{2}{3}$?	$\frac{8}{12}$?
How many eighths in	$\frac{4}{16}$?	$\frac{12}{16}$?	$\frac{8}{16}$?	$\frac{12}{24}$?
How many fifths in	$\frac{8}{10}$?	$\frac{6}{15}$?	$\frac{12}{20}$?	$\frac{16}{20}$?
How many sevenths in	$\frac{10}{14}$?	$\frac{8}{14}$?	$\frac{9}{21}$?	$\frac{12}{21}$?
How many ninths in	$\frac{12}{18}$?	$\frac{16}{18}$?	$\frac{15}{27}$?	$\frac{18}{27}$?
How many tenths in	$\frac{16}{20}$?	$\frac{21}{30}$?	$\frac{24}{40}$?	$\frac{25}{50}$?

To a Common Denominator.

When fractions have the same denominator, they are said to have a *Common Denominator.*

Ex.—Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

Solution.—A common denominator for 3ds and 4ths is 12ths; in 1 there are $\frac{12}{12}$, and in $\frac{2}{3}$ there are $\frac{2}{3}$ of $\frac{12}{12}$, or $\frac{8}{12}$; and in $\frac{3}{4}$ there are $\frac{3}{4}$ of $\frac{12}{12}$, or $\frac{9}{12}$.

Reduce $\frac{1}{4}$ and $\frac{1}{5}$ to a common denominator.

Reduce $\frac{1}{3}$ and $\frac{1}{5}$ to a common denominator.

Addition of Fractions.

What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$?

SOLUTION.—$\frac{2}{3}$ equals $\frac{8}{12}$, and $\frac{3}{4}$ equals $\frac{9}{12}$; $\frac{8}{12}$ plus $\frac{9}{12}$ are $\frac{17}{12}$, which equals $1\frac{5}{12}$.

What is the sum

Of $\frac{1}{4}$ and $\frac{1}{9}$?	Of $\frac{2}{5}$ and $\frac{3}{7}$?
Of $\frac{2}{8}$ and $\frac{3}{5}$?	Of $\frac{5}{6}$ and $\frac{4}{7}$?
Of $\frac{3}{7}$ and $\frac{3}{8}$?	Of $2\frac{1}{2}$ and $3\frac{1}{3}$?
Of $3\frac{1}{3}$ and $4\frac{1}{4}$?	Of $2\frac{2}{3}$ and $1\frac{3}{4}$?
Of $3\frac{1}{4}$ and $2\frac{3}{5}$?	Of $6\frac{2}{5}$ and $5\frac{3}{6}$?
Of $4\frac{1}{6}$ and $5\frac{1}{7}$?	Of $6\frac{1}{5}$ and $5\frac{1}{8}$?
Of $7\frac{2}{3}$ and $8\frac{3}{7}$?	Of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$?
Of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$?	Of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$?

Subtraction of Fractions.

What is the difference between $\frac{3}{4}$ and $\frac{2}{3}$?

SOLUTION.—$\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{2}{3}$ is equal to $\frac{8}{12}$; $\frac{9}{12}$ minus $\frac{8}{12}$ equals $\frac{1}{12}$.

Subtract

$\frac{2}{3}$ from $\frac{4}{5}$?	$\frac{2}{5}$ from $\frac{3}{6}$?	$\frac{3}{5}$ from $\frac{3}{4}$?
$\frac{3}{4}$ from $\frac{8}{9}$?	$\frac{1}{6}$ from $\frac{1}{5}$?	$\frac{1}{7}$ from $\frac{1}{6}$?
$\frac{1}{3}$ from $\frac{5}{8}$?	$\frac{2}{5}$ from $\frac{2}{3}$?	$\frac{1}{2}$ from $\frac{5}{7}$?
$\frac{1}{8}$ from $\frac{1}{7}$?	$\frac{1}{9}$ from $\frac{1}{2}$?	$\frac{2}{7}$ from $\frac{2}{3}$?
$\frac{3}{7}$ from $\frac{6}{8}$?	$2\frac{1}{2}$ from $3\frac{1}{4}$?	$3\frac{1}{5}$ from $4\frac{1}{4}$?
$2\frac{1}{4}$ from $3\frac{1}{6}$?	$3\frac{1}{5}$ from $5\frac{1}{6}$?	

Multiplication of Fractions.

How many are 4 times $\frac{3}{8}$?

SOLUTION.—4 times $\frac{3}{8}$ are $\frac{12}{8}$, which equals $\frac{3}{2}$ or $1\frac{1}{2}$.

How many are 3 times $\frac{3}{6}$?	3 times $\frac{2}{9}$?
How many are 4 times $\frac{6}{8}$?	5 times $\frac{4}{10}$?
How many are 7 times $\frac{3}{14}$?	3 times $\frac{2}{6}$?
How many are 4 times $\frac{6}{12}$?	6 times $\frac{7}{12}$?
How many are 3 times $\frac{6}{9}$?	4 times $\frac{2}{6}$?
How many are 8 times $\frac{2}{4}$?	8 times $\frac{3}{6}$?
How many are 5 times $\frac{3}{6}$?	5 times $\frac{7}{6}$?

Division of Fractions.

How many times is $\frac{2}{3}$ contained in 4?

SOLUTION.—1 contained in 4, 4 times; and if 1 is contained in 4, 4 times, $\frac{1}{3}$ is contained in 4, 3 times 4 times, which are 12 times, and 2 thirds is contained in 4 $\frac{1}{2}$ of 12 times, or 6 times.

How many times is $\frac{3}{4}$ contained in 2? In 3?
How many times is $\frac{2}{5}$ contained in 3? In 5?
How many times is $\frac{3}{7}$ contained in 2? In 4?
How many times is $\frac{2}{3}$ contained in 5? In 7?
How many times is $\frac{5}{6}$ contained in 4? In 5?
How many times is $\frac{6}{8}$ contained in 2? In 4?
How many times is $\frac{2}{4}$ contained in $\frac{3}{5}$?

SOLUTION.—$\frac{2}{4}$ is equal to $\frac{10}{20}$, and $\frac{3}{5}$ is equal to $\frac{12}{20}$; $\frac{10}{20}$ is contained as many times in $\frac{12}{20}$ as 10 is contained 12, which is $\frac{12}{10}$ or $\frac{6}{5}$ times.

How many times is $\frac{3}{4}$ contained in $\frac{2}{5}$?
How many times is $\frac{2}{5}$ contained in $\frac{3}{4}$?
How many times is $\frac{5}{6}$ contained in $\frac{2}{4}$?
How many times $\frac{3}{4}$ is $\frac{3}{2}$? $\frac{5}{6}$? $\frac{3}{6}$?
How many times $\frac{3}{5}$ is $\frac{1}{7}$? $\frac{3}{8}$? $\frac{9}{10}$?

Relation of Fractions.

What part of 2 is $\frac{3}{4}$?

SOLUTION. —1 is $\frac{1}{2}$ of 2; and, if 1 is $\frac{1}{2}$ of 2, $\frac{1}{4}$ is $\frac{1}{4}$ of $\frac{1}{2}$, which is $\frac{1}{8}$ of 2, and $\frac{3}{4}$ is 3 times $\frac{1}{8}$, or $\frac{3}{8}$ of 2.

What part of 3 is $\frac{2}{3}$? Of 2 is $\frac{3}{4}$?
What part of 4 is $\frac{2}{5}$? Of 5 is $\frac{3}{5}$?
What part of 4 is $\frac{4}{6}$? Of 7 is $\frac{5}{8}$?
What part of 9 is $\frac{3}{7}$? Of 5 is $\frac{1}{2}$ of $\frac{3}{4}$?
What part of 6 is $\frac{2}{3}$ of $\frac{2}{4}$? Of 7 is $\frac{2}{5}$ of $\frac{3}{2}$?
What part of $\frac{2}{3}$ is $\frac{4}{5}$?

SOLUTION.—$\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$, and $\frac{3}{3}$, or *one*, is 3 times $\frac{1}{2}$, or $\frac{3}{2}$ of $\frac{2}{3}$. Since *one* is $\frac{3}{2}$ of $\frac{2}{3}$, $\frac{1}{5}$ is $\frac{1}{5}$ of $\frac{3}{2}$, which is $\frac{3}{10}$ of $\frac{2}{3}$, and $\frac{4}{5}$ is 4 times $\frac{3}{10}$, which are $\frac{12}{10}$, or $\frac{6}{5}$ of $\frac{2}{3}$.

What part of $\frac{3}{4}$ is $\frac{2}{5}$? Of $\frac{3}{6}$ is $\frac{2}{3}$? Of $\frac{4}{5}$ is $\frac{5}{6}$?
What part of $\frac{6}{7}$ is $\frac{3}{8}$? Of $\frac{2}{5}$ is $\frac{3}{4}$? Of $\frac{3}{5}$ is $\frac{3}{4}$?

What is $\frac{1}{3}$ of 4?

SOLUTION.—$\frac{1}{3}$ of 1 is $\frac{1}{3}$, and if $\frac{1}{3}$ of 1 is $\frac{1}{3}$, $\frac{1}{3}$ of 4 is 4 times $\frac{1}{3}$, which are $\frac{4}{3}$.

What is

$\frac{1}{3}$ of 5?	$\frac{1}{4}$ of 6?	$\frac{1}{5}$ of 7?	$\frac{1}{6}$ of 9?
$\frac{1}{2}$ of 5?	$\frac{1}{4}$ of 10?	$\frac{1}{7}$ of 12?	$\frac{1}{8}$ of 20?
$\frac{1}{10}$ of 24?	$\frac{1}{12}$ of 32?	$\frac{2}{4}$ of 6?	$\frac{3}{4}$ of 10?

What is $\frac{2}{3}$ of $\frac{6}{8}$?

SOLUTION.—$\frac{1}{3}$ of $\frac{6}{8}$ is $\frac{2}{8}$, and if $\frac{1}{3}$ of $\frac{6}{8}$ is $\frac{2}{8}$, $\frac{2}{3}$ of $\frac{6}{8}$ are 2 times $\frac{2}{8}$, which are $\frac{4}{8}$, or $\frac{1}{2}$.

What is

$\frac{3}{5}$ of $\frac{10}{12}$?	$\frac{2}{7}$ of $\frac{7}{8}$?	$\frac{3}{6}$ of $\frac{6}{9}$?	$\frac{4}{5}$ of $\frac{18}{20}$?
$\frac{3}{5}$ of $\frac{15}{18}$?	$\frac{2}{7}$ of $\frac{14}{16}$?	$\frac{2}{3}$ of $\frac{9}{12}$?	

What is $\frac{1}{3}$ of $\frac{1}{4}$?

SOLUTION.—$\frac{1}{4}$ is one of the 4 equal parts into which a unit may be divided; if we divide each *fourth* into 3 equal parts, each part is $\frac{1}{3}$ of $\frac{1}{4}$, and since there are 4 times 3, or 12 parts in all, each part is $\frac{1}{12}$ of a unit?

ANOTHER SOLUTION.—$\frac{1}{4}$ equals $\frac{3}{12}$, and $\frac{1}{3}$ of $\frac{3}{12}$ is $\frac{1}{12}$. This is a simpler solution, but not so explanatory.

What is $\frac{1}{2}$ of $\frac{1}{4}$?	$\frac{1}{4}$ of $\frac{1}{4}$?	$\frac{1}{5}$ of $\frac{1}{4}$?
What is $\frac{1}{3}$ of $\frac{1}{5}$?	$\frac{1}{2}$ of $\frac{1}{3}$?	$\frac{1}{3}$ of $\frac{1}{6}$?
What is $\frac{1}{4}$ of $\frac{1}{6}$?	$\frac{1}{5}$ of $\frac{1}{6}$?	$\frac{1}{6}$ of $\frac{1}{7}$?
What is $\frac{1}{5}$ of $\frac{1}{3}$?	$\frac{1}{2}$ of $\frac{1}{7}$?	$\frac{1}{3}$ of $\frac{1}{7}$?
What is $\frac{1}{4}$ of $\frac{1}{8}$?	$\frac{1}{6}$ of $\frac{1}{8}$?	$\frac{1}{7}$ of $\frac{1}{9}$?
What is $\frac{1}{6}$ of $\frac{1}{12}$?	$\frac{1}{8}$ of $\frac{1}{9}$?	$\frac{1}{7}$ of $\frac{1}{10}$?
What is $\frac{1}{9}$ of $\frac{1}{11}$?	$\frac{1}{10}$ of $\frac{1}{11}$?	$\frac{1}{11}$ of $\frac{1}{12}$?

What is $\frac{2}{3}$ of $\frac{4}{5}$?

SOLUTION.—$\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, and if $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times $\frac{1}{15}$, which are $\frac{4}{15}$, and $\frac{2}{3}$ of $\frac{4}{5}$ are 2 times $\frac{4}{15}$, or $\frac{8}{15}$.

What is $\frac{1}{3}$ of $\frac{2}{3}$?	$\frac{1}{4}$ of $\frac{3}{5}$?	$\frac{1}{5}$ of $\frac{4}{6}$?
What is $\frac{1}{4}$ of $\frac{5}{7}$?	$\frac{1}{6}$ of $\frac{5}{8}$?	$\frac{1}{7}$ of $\frac{5}{6}$?
What is $\frac{1}{5}$ of $\frac{6}{7}$?	$\frac{1}{8}$ of $\frac{7}{9}$?	$\frac{1}{6}$ of $\frac{4}{12}$?
What is $\frac{2}{3}$ of $\frac{5}{6}$?	$\frac{3}{4}$ of $\frac{5}{6}$?	$\frac{4}{5}$ of $\frac{6}{8}$?
What is $\frac{3}{5}$ of $\frac{4}{6}$?	$\frac{5}{6}$ of $\frac{5}{7}$?	$\frac{3}{4}$ of $\frac{3}{6}$?

CONTRACTIONS

IN

FRACTIONS.

To square any number containing $\frac{1}{2}$, as $7\frac{1}{2}$, $8\frac{1}{2}$.

RULE.—Multiply the whole number by the next higher whole number, and annex $\frac{1}{4}$ to the product.

Ex. 1. What is the square of $7\frac{1}{2}$? Ans. $56\frac{1}{4}$.

We simply say, 7 times 8 are 56, to which we add $\frac{1}{4}$.

2. What will $9\frac{1}{2}$ lbs. beef cost at $9\frac{1}{2}$ cts. a lb.?
3. What will $12\frac{1}{2}$ yds. tape cost at $12\frac{1}{2}$ cts. a yd.?
4. What will $5\frac{1}{2}$ lbs. nails cost at $5\frac{1}{2}$ cts. a lb.?
5. What will $11\frac{1}{2}$ yds. tape cost at $11\frac{1}{2}$ cts. a yd.?
6. What will $19\frac{1}{2}$ bu. bran cost at $19\frac{1}{2}$ cts. a bu.?

REASON.—We multiply the whole number by the next higher whole number, because half of any number taken twice and added to its square is the same as to multiply the given number by ONE more than itself. The same principle will multiply any two *like* numbers together, when the sum of the fractions is ONE, as $8\frac{1}{3}$ by $8\frac{2}{3}$, or $11\frac{3}{8}$ by $11\frac{5}{8}$, etc. It is obvious

that to multiply any number by any two fractions whose sum is ONE, that the sum of the products *must be the original number*, and adding the number to its square is simply to multiply it by ONE more than itself; for instance, to multiply $7\frac{1}{4}$ by $7\frac{3}{4}$, we simply say, 7 times 8 are 56, and then, to complete the multiplication, we add, of course, the product of the fractions ($\frac{3}{4}$ times $\frac{1}{4}$ are $\frac{3}{16}$), making $56\frac{3}{16}$ the answer.

Multiplication,

WHERE THE SUM OF FRACTIONS IS ONE.

To multiply any two *like* numbers together when the sum of the fractions is ONE.

RULE.—Multiply the whole number by the next higher whole number; after which, add the product of the fractions.

Multiply $3\frac{3}{4}$ by $3\frac{1}{4}$ in a single line.

Multiply $\frac{1}{4} \times \frac{3}{4}$, which gives $\frac{3}{16}$, and set down the result; then we multiply the 3 in the multiplicand, increased by unity, by the 3 in the multiplier, 3×4, which gives 12 and completes the product.

$$\begin{array}{r} 3\frac{3}{4} \\ 3\frac{1}{4} \\ \hline 12\frac{3}{16} \end{array}$$

Multiply $7\frac{2}{5}$ by $7\frac{3}{5}$ in a single line.

Multiply $\frac{3}{5} \times \frac{2}{5}$, which gives $\frac{6}{25}$, and set down the result; then we multiply the 7 in the multiplicand, increased by unity, by the 7 in the multiplier, 7×8, which gives 56 and completes the product.

$$\begin{array}{r} 7\frac{2}{5} \\ 7\frac{3}{5} \\ \hline 56\frac{6}{25} \end{array}$$

Multiply $11\frac{1}{3}$ by $11\frac{2}{3}$ in a single line.

Multiply $\frac{2}{3} \times \frac{1}{3}$, which gives $\frac{2}{9}$, and set down the result; then we multiply the 11 in the multiplicand, increased by unity, by the 11 in the multiplier, 11×12, which gives 132, and completes the product.

$$\begin{array}{r} 11\frac{1}{3} \\ 11\frac{2}{3} \\ \hline 132\frac{2}{9} \end{array}$$

CONTRACTIONS IN FRACTIONS.

To multiply any two numbers together, each of which involves the fraction $\frac{1}{2}$, as $7\frac{1}{2}$ by $9\frac{1}{2}$, etc.

RULE.—To the product of the whole numbers add half their sum plus $\frac{1}{4}$.

1. What will $3\frac{1}{2}$ doz. eggs cost at $7\frac{1}{2}$ cts. a doz.?

Here the sum of 7 and 3 is 10, and half this sum is 5, so we simply say, 7 times 3 are 21 and 5 are 26, to which we add $\frac{1}{4}$.

$$\begin{array}{r} 3\frac{1}{2} \\ 7\frac{1}{2} \\ \hline 26\frac{1}{4} \end{array}$$

N. B. If the sum be an odd number, call it one less to make it even, and in such cases the fraction must be $\frac{3}{4}$.

2. What will $11\frac{1}{2}$ lbs. cheese cost at $9\frac{1}{2}$ cts. a lb.?
3. What will $8\frac{1}{2}$ yds. tape cost at $15\frac{1}{2}$ cts. a yd.?
4. What will $7\frac{1}{2}$ lbs. rice cost at $13\frac{1}{2}$ cts. a lb.?
5. What will $10\frac{1}{2}$ bu. coal cost at $12\frac{1}{2}$ cts. a bu.?

REASON.—In explaining the above rule, we add half their sum, because half of either number added to half the other would be half their sum, and we add $\frac{1}{4}$, because $\frac{1}{2}$ by $\frac{1}{2}$ is $\frac{1}{4}$. The same principle will multiply any two numbers together, each of which has the same fraction; for instance, if the fraction was $\frac{1}{3}$, we would add one-third their sum; if $\frac{3}{4}$, we would add three-fourths their sum, etc.; and then, to complete the multiplication, we would add, of course, the product of the fractions.

MULTIPLYING ANY TWO NUMBERS TOGETHER, EACH INVOLVING THE SAME FRACTION.

RULE.—To the product of the whole numbers, add the product of their sum by either fraction; after which, add the product of their fractions.

1. What will $11\frac{3}{4}$ lbs. rice cost at $9\frac{3}{4}$ cts. a lb.?

Here the sum of 9 and 11 is 20, and three-fourths of this sum is 15, so we simply say, 9 times 11 are 99 and 15 are 114, to which we add the product of the fractions ($\frac{9}{16}$).

$$\begin{array}{r} 11\frac{3}{4} \\ 9\frac{3}{4} \\ \hline 114\frac{9}{16} \end{array}$$

2. What will $7\frac{2}{3}$ doz. eggs cost at $8\frac{2}{3}$ cts. a doz.?
3. What will $6\frac{3}{4}$ bu. coal cost at $6\frac{3}{4}$ cts. a bu.?
4. What will $45\frac{3}{4}$ bu. seed cost at $3\frac{3}{4}$ dols. a bu.?
5. What will $3\frac{3}{8}$ yds. cloth cost at $5\frac{3}{8}$ dols. a yd.?
6. What will $17\frac{2}{5}$ ft. boards cost at $13\frac{2}{5}$ cts. a ft.?
7. What will $18\frac{3}{4}$ lbs. butter cost at $18\frac{3}{4}$ cts. a lb.?

N. B. If the product of the sum by either fraction is a whole number with a fraction, it is better to reserve the fraction until we are through with the whole numbers, and then add it to the product of the fractions; for instance, to multiply $3\frac{1}{4}$ by $7\frac{1}{4}$, we find the sum of 7 and 3, which is 10, and one-fourth of this sum is $2\frac{1}{2}$; setting the $\frac{1}{2}$ down in some waste spot, we simply say, 7 times 3 are 21 and 2 are 23; then, adding the $\frac{1}{2}$ to the product of the fractions ($\frac{1}{16}$), gives $\frac{9}{16}$, making $23\frac{9}{16}$, Ans.

MULTIPLYING ANY MIXED NUMBERS.

RULE 1.—Multiply the whole numbers together.
2.—Multiply the upper digit by the lower fraction.
3.—Multiply the lower digit by the upper fraction.
4.—Multiply the fractions together.
5.—Add these *four* products together.

	$12\frac{2}{3}$
EXAMPLE.—Multiply $12\frac{2}{3}$ by $9\frac{3}{4}$.	$9\frac{3}{4}$
1.—We multiply the whole numbers =	108
2.—Multiply 12 by $\frac{3}{4}$ =	9
3 —Multiply 9 by $\frac{2}{3}$ =	6
4.—Multiply $\frac{3}{4}$ by $\frac{2}{3}$ =	$0\frac{6}{12}$
5.—Add these *four* products together,	$123\frac{6}{12}$

and we have the complete result.

N. B. When the student has become familiar with the above process it is better to do the intermediate work mentally, and, instead of writing out the partial products, add them in the mind as you pass along, and thus proceed very rapidly.

PRACTICAL BUSINESS METHOD

FOR MULTIPLYING ALL MIXED NUMBERS.

Business men generally, in multiplying the mixed numbers, only care about having the answer correct to the nearest cent; that is, they disregard the fraction. When it is a half cent or more, they call it another cent; if less than half a cent, they drop it. And the object of the following rule is to show the easiest and most rapid process of finding the product to the nearest unit of any two numbers, one or both of which involves a fraction.

Multiply $8\frac{1}{5}$ by $10\frac{1}{4}$.

Here we simply say 10 times 8 are 80 and $\frac{1}{4}$ of 8 is 2, making 82, and $\frac{1}{5}$ of 10 is 2, which makes 84; then $\frac{1}{4}$ times $\frac{1}{5}$ is $\frac{1}{20}$, making $84\frac{1}{20}$ the answer.

$$\begin{array}{r} 8\frac{1}{5} \\ 10\frac{1}{4} \\ \hline 84\frac{1}{20} \end{array}$$

TO MULTIPLY ANY TWO NUMBERS TO THE NEAREST UNIT

General Rule 1.—Multiply the whole number in the multiplicand by the fraction in the multiplier to the nearest unit.

2.—Multiply the whole number in the multiplier by the fraction in the multiplicand to the nearest unit.

3.—Multiply the whole numbers together and add the three products in your mind as you proceed.

N. B. In actual business the work can generally be done mentally; only easy fractions *occur in business*

N. B. This rule is so simple and so true, according to all business usage, that every accountant should make himself perfectly familiar with its application. There being no such thing as a fraction to add in, there is scarcely any liability to error or mistake. By no other arithmetical process can the result be obtained by so few figures.

EXAMPLES FOR MENTAL OPERATION.

EXAMPLE FIRST.

Multiply $11\frac{1}{3}$ by $8\frac{1}{4}$ by business method.

Here $\frac{1}{4}$ of 11 to the nearest unit is 3, and $\frac{1}{3}$ of 8 to the nearest unit is 3, making 6, so we simply say, 8 times 11 are 88 and 6 are 94, Ans.

$$\begin{array}{r} 11\frac{1}{3} \\ 8\frac{1}{4} \\ \hline 94 \end{array}$$

REASON.—$\frac{1}{4}$ of 11 is nearer 3 than 2, and $\frac{1}{3}$ of 8 is nearer 3 than 2. Make the nearest whole number the quotient.

EXAMPLE SECOND.

Multiply $7\frac{3}{4}$ by $9\frac{2}{5}$ by business method.

Here $\frac{2}{5}$ of 7 to the nearest unit is 3, and $\frac{3}{4}$ of 9 to the nearest unit is 7; then 3 plus 7 is 10, so we simply say, 9 times 7 are 63 and 10 are 73, Ans.

$$\begin{array}{r} 7\frac{3}{4} \\ 9\frac{2}{5} \\ \hline 73 \end{array}$$

EXAMPLE THIRD.

Multiply $23\frac{1}{3}$ by $19\frac{1}{4}$ by business method.

Here $\frac{1}{4}$ of 23 to the nearest unit is 6, and $\frac{1}{3}$ of 19 to the nearest unit is 6; then 6 plus 6 is 12, so we simply say, 19 times 23 are 437 and 12 are 449, Ans.

$$\begin{array}{r} 23\frac{1}{3} \\ 19\frac{1}{4} \\ \hline 449 \end{array}$$

EXAMPLE FOURTH.

Multiply $128\frac{2}{3}$ by 25 by business method.

Here $\frac{2}{3}$ of 25 to the nearest unit is 17, so we simply say, 25 times 128 are 3200 and 17 are 3217, the answer.

$$\begin{array}{r} 128\frac{2}{3} \\ 25 \\ \hline 3217 \end{array}$$

PRACTICAL EXAMPLES FOR BUSINESS MEN.

1 What is the cost of $17\frac{1}{2}$ lbs. sugar at $18\frac{3}{4}$ cts. per lb.?

Here $\frac{3}{4}$ of 17 to the nearest unit is 13, and $\frac{1}{2}$ of 18 is 9, 13 plus 9 is 22, so we simply say, 18 times 17 are 306 and 22 are 328, the answer.

$$\begin{array}{r} 17\frac{1}{2} \\ 18\frac{3}{4} \\ \hline \$3.28 \end{array}$$

2. What is the cost of 11 lbs. 5 oz. of butter at $33\frac{1}{3}$ cts. per lb.?

Here $\frac{1}{3}$ of 11 to the nearest unit is 4, and $\frac{5}{16}$ of 33 to the nearest unit is 10; then 4 plus 10 is 14, so we simply say, 33 times 11 are 363 and 14 are 377, Ans.

$$\begin{array}{r} 11\frac{5}{16} \\ 33\frac{1}{3} \\ \hline \$3.77 \end{array}$$

3. What is the cost of 17 doz. and 9 eggs at $12\frac{1}{2}$ cts. per doz.?

Here $\frac{1}{2}$ of 17 to the nearest unit is 9, and $\frac{9}{12}$ of 12 is 9; then nine plus 9 is 18, so we simply say, 12 times 17 are 204 and 18 are 222, the answer.

$$\begin{array}{r} 17\frac{9}{12} \\ 12\frac{1}{2} \\ \hline \$2.22 \end{array}$$

4. What will be the cost of $15\frac{3}{4}$ yds. calico at $12\frac{1}{2}$ cts. per yd.? Ans. \$1.97.

DECIMALS.

To reduce a decimal to a common fraction.

Rule.—Write the decimal as it stands, omitting the decimal point, for the numerator. For the denominator write 1 with as many ciphers annexed as there are decimal places in the numerator.

EXAMPLES.

Reduce .25 to an equivalent common fraction.

Ans. $\frac{25}{100}$, which reduced to its lowest terms $= \frac{1}{4}$.

Reduce .375 to a common fraction. Ans. $\frac{375}{1000} = \frac{3}{8}$.

Reduce .1875 to a common fraction. Ans. $\frac{3}{16}$.

Reduce .625 to a common fraction. Ans. $\frac{5}{8}$.

To reduce common fractions to decimals.

Rule.—Annex ciphers to the numerator, and divide by the denominator, prefixing a point to the quotient. There must be as many places in the quotient as there have been ciphers annexed; if not enough, prefix ciphers.

EXAMPLES.

Reduce $\frac{3}{4}$ to a decimal.

$$\begin{array}{r} 4)\underline{3.00} \\ .75 = \frac{75}{100} \text{ Ans.} \end{array}$$

Reduce $\frac{3}{8}$ to a decimal. Ans. .375.

" $\frac{6}{7}$ " " Ans. .8571+.

" $\frac{1}{4}$ " "

" $\frac{2}{3}$ " "

MULTIPLICATION OF DECIMALS.

RULE.—Multiply as in whole numbers, and from the right of the product point off as many figures for decimals as there are decimal places in both *multiplicand* and *multiplier*.

If there are not figures enough in the product, prefix ciphers.

EXAMPLES.

Multiply 4.25 by 6.5.

$$\begin{array}{r} 4.25 \\ \underline{6.5} \\ 2125 \\ \underline{2550} \\ 27.625 \text{ Ans.} \end{array}$$

Multiply 84.5 by 4.

$$\begin{array}{r} 84.5 \\ \underline{4} \\ 338.0 \end{array}$$

Multiply 6.425 by 4.25.

" 18.5 " 6.75.

" 12.575 " 9.375.

" .25 " 6.0025.

" .275 " 3.0025.

" 18.625 " 5.25.

What is the cost of $12\frac{3}{4}$ lbs. at $6\frac{1}{4}$ cts. per lb?

What is the cost of $7\frac{2}{5}$ yds. at $18\frac{3}{4}$ cts. per yd?

NOTE.—It is sometimes more convenient to change common fractions to decimals before multiplying. $18\frac{3}{4} \times 12\frac{1}{2} = 18.75 \times 12.5$.

UNITED STATES MONEY.

10 Mills (m)	= 1 cent,	ct.
10 Cents	= 1 dime,	d.
10 Dimes	= 1 dollar,	$.
10 Dollars	= 1 eagle,	E.

The origin of the symbol $, or the United States dollar mark, has been ascribed to several sources. By some it is supposed to represent the *U* written upon the *S*, denoting U. S. (United States). Some think it is a modification of the figure 8, having reference to 8 reals, or piece of Eight, as the dollar was formerly called; others, that it represents the "Pillars of Hercules," which were stamped on the Pillar Dollar; and others, still, that it is a combination of the initials P. and S., from the Spanish *Peso Duro*, signifying Hard Dollar. As it is used in Portugal to note the thousands' place, it is probable that it originated in that country: a Mil-reis, or thousand reis, is written thus, 1$000.

The term Dime is from the French *disme*, meaning ten.

The term Cent is from the Latin *centum*, a hundred.

The term Mill is from the Latin *mille*, a thousand.

UNITED STATES COINS.

The Gold Coins are the Double Eagle, $20.00; Eagle, $10.00; Half Eagle, $5.00; Quarter Eagle, $2.50; three dollar piece and dollar.

The Fifty Dollar Piece is not a legal coin. The Half Copper Cent is no longer coined. The Mill is not a coin. Gold coin contains nine parts gold and one part copper and silver.

The Silver Coins are Dollar, Half Dollar, Quarter Dollar, Dime, Half Dime and Three Cent Piece. Silver coins contain 9 parts silver and 1 part copper.

The Nickel Coins are the Cent the new Three Cent Piece and new Five Cent Piece.

The Nickel contains 88 parts copper and 12 parts nickel.

The Copper Coins are the Cent and Two Cent Pieces. The Two Cent and Cent Pieces are made of nickel and copper.

One Eagle (Gold) weighs 258 troy grains.

One Dollar (Silver) weighs 412.5 troy grains.

One Cent (Copper) weighs 168 troy grains.

23.2 grains of pure gold = $1.00.

Gold Coins prior to 1834, like that of England = 88.8 per dwt. By an act of Congress of 1834 its value was made 94.8 cents per dwt. The old U.S. Eagle coined previous to 1834 is worth $10.66.8.

By an act of Congress the payment of debt with coin is regulated as follows:

All Gold Coins at their respective value for any amount. The Half Dollar, Quarter Dollar, Dime and Half Dime at their respective value for debt under $5.00. The Three Cent Piece for debts of any amount under thirty cents. The one cent pieces for debts of any amount under 10 cents.

STATE CURRENCY.

The money of this country before the adoption of the decimal currency by Congress in 1786 was in the denominations of pounds, shillings, and pence. The *Colonial notes* which were then in circulation had depreciated in value; and the number of shillings equivalent to a dollar at that time are given in the following table:—

New England Currency.

New England States, Virginia, Kentucky, and Tennessee,	$1 = 6*s*. = 72*d*. 1*s*. = 16⅔ cts.

New York Currency.

New York, Ohio, Michigan, and North Carolina,	$1 = 8*s*. = 96*d*. 1*s*. = 12½ cts.

Pennsylvania Currency.

Pennsylvania, New Jersey, Delaware, and Maryland,	$1 = 7*s*. 6*d*. = 90*d*. 1*s*. = 13⅓ cts.

Georgia Currency.

Georgia and South Carolina,	$1 = 4*s*. 8*d*. = 56*d*. 1*s*. = 21$\frac{3}{7}$ cts.

The Coins of Foreign Nations,

With their value in United States coins, as determined by the recognized standard at the Mint in Philadelphia.

GOLD COINS.

COUNTRY.	DENOMINATION.	Value in U. S. money $	cts.	m.
Austria	Fourfold ducat	9	13	1
"	4 florins (new)	1	93	5
"	Ducat	2	27	9
Belgium	25 francs	4	72	0
Brazil	29 milreis	10	89	3
Central America	2 escudos	3	68	7
"	4 reals	0	48	7
Chili	10 pesos (dollar)	9	11	4
Columbia and South America generally	Old doubloon	15	59	2
Columbia	20 pesos, "Bogota"	18	94	6
"	20 pesos, "Medellin"	19	05	5
"	20 pesos, "Popayan"	18	96	2
Costa Rica	10 pesos	8	44	7
Denmark	20 crown	5	35	8
"	Old ten thaler	7	89	8
Egypt	Bedidlik (100 piasters)	4	97	3
England	Pound, or sovereign (new)	4	86	5
France	20 francs	3	84	5
German Empire	New 20 marks	4	76	3
Greece	20 drachms	3	44	2
India (British)	Mohur, or 15 rupees	7	10	3
Italy	20 lire	3	84	5
Japan	20 yen	19	94	2
Mexico	Doubloon	15	60	0
Netherlands	10 gilders	3	99	3
New Grenada	10 pesos (dollars)	9	67	4
Norway	20 crowns	5	35	8
Peru	20 soles	19	23	7
Portugal	Coroa (crown)	5	80	5
Russia	5 rubles	3	97	5
Spain	100 reals	4	96	3
"	80 reals	3	86	2
Sweden	Ducat	2	23	6
"	Carolin (10 francs)	1	93	4
Tunis	25 piasters	2	99	5
Turkey	100 piasters	4	36	8

SILVER COINS.

Country.	Denomination.	Value in U. S. money $	cts.	m.
Austria	Old rix-dollar	0	95	4
"	New florin	0	45	3
"	New Union dollar	0	68	1
Belgium	5 francs	0	91	5
Bolivia	New dollar	0	91	5
Brazil	Double milreis	0	95	4
Canada	20 cents	0	17	6
Central America	Dollar	0	93	5
Chili	Old dollar	0	99	6
"	New dollar	0	91	6
China	Dollar (English mint)	0	99	1
"	10 cents	0	09	9
Denmark	2 rigsdaler	1	03	2
Egypt	Piaster (new)	0	03	8
England	Shilling (new)	0	21	4
"	Shilling (average)	0	20	9
"	Florin	0	42	9
France	5 francs	0	91	6
North German States	Thaler, before 1857	0	67	8
" " "	Thaler (new)	0	68	0
South German States	Florin	0	38	9
German Empire	5 marks (new)	0	91	9
Greece	5 drachms	0	82	2
Hindostan	Rupee	0	43	4
Italy	5 lire	0	91	7
Japan	1 yen	0	99	1
"	50 sen	0	40	8
Mexico	Dollar	0	99	8
Netherlands	2½ gilders	0	96	4
Norway	Specie daler	1	93	2
New Grenada	Dollar of 1857	0	91	1
Peru	Old dollar	0	99	0
Portugal	500 reis	0	46	3
Roumania	2 lei (francs), new	0	34	1
Russia	Ruble	0	74	1
Spain	5 pesetas (dollars)	0	91	4
Sweden	Riksdaler	0	26	0
Switzerland	2 francs	0	33	9
Tunis	5 piasters	0	58	3
Turkey	20 piasters	0	81	1

GREAT BRITAIN'S MONEY.

4 Farthings	= 1 Penny,	*d.*
12 pence	= 1 Shilling,	*s.*
20 shillings	= 1 Pound,	£.

The *Gold coins* are the sovereign, which represents the pound, and the half-sovereign. The guinea, of 21 shillings, and its subdivisions, have not been coined since 1816.

The *Silver coins* are crowns of 5*s*., half-crowns, florins of 2*s*., shillings, the 6*d*, the 4*d*. or groats, and 3*d* pieces.

The *Copper coins* are the penny, half-penny, and farthing, coined at the rate of 24 pence per pound avoirdupois

Bank of England Notes are a legal tender for any sum over £5; silver is not a legal tender over 40*s*.; copper, for not more than 12*d*. in pennies or half-pennies; or 6*d*., in farthings.

£ is a contraction of *librae*, *s*. of *solidi*, *d*. of *denarii*, and *q*. of *quadrantes*; farthing is another word for *fourthing*.

The word sterling is supposed to be derived from the first coiners of English silver, who came into England from Germany in the reign of Richard I., and were called *Easterlings*. It is used to distinguish the currency of Great Britain from that of the Colonies, and from some continental money bearing the same denominations.

Valuable Information for Business Men.

NOTES.

Demand Notes are payable on presentation, without grace, and bear legal interest after a demand has been made, if not so written. An endorser on a demand note is holden only for a limited time, variable in different States.

A Negotiable Note must be made payable either to bearer, or be properly endorsed by the person to whose order it is made. If the endorser wishes to avoid responsibility, he can endorse "without recourse."

A Joint Note is one signed by two or more persons, who can each become liable for the whole amount.

Three Days' Grace are allowed on all time notes, after the time for payment expires; if not then paid, the endorser, if any, should be legally notified to be holden.

Notes Falling Due Sunday, or on a legal holiday, must be paid the day previous.

Notes Dated Sunday are void.

Altering a Note in any manner, by the holder, makes it void.

Notes Given by Minors are void.

The Maker of a Note that is lost or stolen is not released from payment if the amount and consideration can be proven.

Notes Obtained by Fraud, or given by intoxicated persons, cannot be collected.

An Endorser has a right of action against all whose names were previously on a note endorsed by him.

BILLS OF EXCHANGE, DRAFTS, ACCEPTANCES.

A Bill of Exchange, or Draft is an order drawn by one person, or firm, upon another, payable either at sight or at a stated future time.

It becomes an "Acceptance" when the party upon whom it is drawn writes across the face "Accepted," and signs his name thereto; and is negotiable and bankable the same as a note, and is subject to the same laws.

In many States both Sight and Time Drafts are entitled to three days' grace, the same as notes; but if made in form of bank check, "pay to," without the words "at sight," it is payable on presentation, without grace.

HOW TO ENDORSE A CHECK.

Very few otherwise intelligent and educated people understand how to properly endorse a Bank check payable to their order, and few realize the inconvenience they cause, by placing their endorsement in an awkward position. An observance of the following rules will enable anyone to place their signature in the proper place.

1. Write *across* the back—not lengthwise.

2. The top of the *back* is the *left* end of the *face.*

3. To deposit a check, write "For Deposit," and below this your name. A clerk not having power of attorney to sign or indorse checks, can deposit his firm's checks by writing on the top of the back "For deposit only to credit of ______[FIRM'S NAME]______ and below this write his own name.

4. Simply writing your name on the back of a check signifies that it has passed through your hands, and is payable to bearer.

5. Always indorse a check just as it appears on the face. For instance, if the check is payable to "G. Read," indorse "G. Read;" if to "Geo. Read," indorse "Geo. Read;" if to "George F. Read," indorse "George F. Read." If the spelling of the name on the face of the check is wrong, indorse first just as the face appears, and below, the proper way. For instance, the check is payable on face to "George Reade;" indorse "George Reade," and below this first indorsement write what it shouid have been, "George Read."

6. If you wish to make the check payable to some particular person, write, "Pay to ___[PERSON'S NAME]___ or order." ___[YOUR NAME.]___

NOTE. In England all checks are payable to bearer; but in this country all strangers presenting checks for payment must be identified by some one known to the bank.

CALCULATIONS

USED IN PARTICULAR BRANCHES OF BUSINESS.

To find the value of tons and hundred-weight without the use of fractions?

Rule. — Multiply the number of hundred-weight by 5, and annex the product to the tons, as so many hundredths of tons; then multiply by the given price per ton, and point off two decimals.

EXAMPLES.

1. What is the cost of 18 tons, 17 cwt. coal at $4 per ton?

$17 \times 5 = 85 \qquad 18.85 \times 4 = 75.40.$ Ans. $75.40.

2. What is the cost of 35 tons, 15 cwt. hay at $12 per ton?

3. What is the cost of 48 tons, 17 cwt. coal at $6.50 per ton?

To find the value of shillings and pence in the decimals of a pound sterling.

Rule.—Multiply the shillings by 5, and call the product hundredths.

Multiply the pence by $4\frac{1}{6}$, and call the product thousandths.

The sum of these two values will be the decimal required.

EXAMPLES.

1. Reduce 12*s*. 6*d*. to the decimal of a pound.

$$\begin{aligned} 12 \times 5 &= .60 \\ 6 \times 4\tfrac{1}{6} &= .025 \\ \hline &\ .625 \end{aligned}$$

2. Reduce £187 13*s*. 3*d*. to the decimal of a pound.

$$
\begin{array}{rl}
13 \times 5 & = .65 \\
3 \times 4\frac{1}{6} & = \underline{.0125} \\
 & .6625 \quad \text{Ans. £187.6625.}
\end{array}
$$

To change aunes to yards.

NOTE.—An aune is a French measure, equal to 1¼ yards.

RULE.—Annex a cipher and divide by 8.

EXAMPLES.

1. In 484 aunes, how many yards?

$$4840 \div 8 = 605 \quad \text{Ans.}$$

2. In 3848 aunes, how many yards? In 1265? In 1847?

NOTE.—$1\frac{1}{4} = \frac{5}{4}$, or $\frac{10}{8}$. This rule can easily be applied to numerous other calculations. The contents of boards $1\frac{1}{4}$ inches thick, etc., may be computed in this manner; the selling price of goods in order to gain 25 per cent on the cost, and others.

3. What is the selling price of goods, which cost 64 cents per yard, to gain 25 per cent?

$$640 \div 8 = 80 \qquad \begin{array}{l} 64 \\ \underline{16} = 25\ \% \text{ of } 64. \\ 80 \end{array}$$

To find how many gallons of linseed oil in a given number of pounds, at $7\frac{1}{2}$ lbs. per gallon.

RULE.—Add one-third of the number of pounds to itself, and point off one decimal.

EXAMPLES.

1. How many gallons in 675 lbs.?

$$\begin{array}{l} 675 \\ \underline{225} = \frac{1}{3} \text{ of } 675 \\ 90.0 \end{array} \qquad \text{Ans. 90 gals.}$$

2. In 1846 lbs. how many gallons? In 675? In 339 lbs.?

Another Method

To reduce pounds, shillings and pence to the decimal of a pound.

Rule. -- Write one-half of the greatest even number of shillings as tenths, and if there be an odd shilling write five hundredths; reduce the pence and farthings to farthings, and write their number as thousandths. If the number of farthings is between 12 and 36, add one to the thousandths; if between 36 and 48, add 2 to the thousandths.

EXAMPLE:—

£3 14s. 6 = £3.725.

Put down the £3, then divide the 14 by 2 and put down 7, then multiply 6 by 4 and add 1 to get the 25. Again,

£4 15s. 10d. = £4 - $7\frac{5}{4}2$ = £4.792.

In this example one-half of 15 is $7\frac{1}{2}$; therefore we put it down in the decimal form .75, and 4 times 10 are 40, add 2 and we have 42, which added to .75 gives .792.

EXERCISES.

1. £2 3s. 6d.	6. £3 3s. 2d.	11. £5 12s. 11d.
2. £4 6s. 5d.	7. £2 5s. 3d.	12. £9 15s. 10d.
3. £8 10s. 3d.	8. £4 7s. 4d.	13. £8 17s. 9d.
4. £9 4s. 5d.	9. £6 9s. 5d.	14. £6 18s. 8d.
5. £10 2s. 3d.	10. £1 11s. 6d.	15. £5 16s. $7\frac{1}{2}$d.

To reduce pounds to dollars.

Rule. -- Multiply the number of pounds by $4.84.

EXAMPLE:—

Reduce £3.725 to dollars and cents.

3.725 X 4.84 = 18.029.

INTEREST.

Interest is the money which is paid for the use of money.

The ***Principal*** is the sum for the use of which interest is paid.

The ***Rate*** is the per cent. of the principal paid for any given time.

NOTE. — When no time is mentioned, *per annum*, or *by the year*, is understood.

The ***Amount*** is the sum of the principal and interest.

Simple Interest is the interest on the sum loaned for the given time, at the given rate.

Legal Interest is the interest according to a certain rate per annum, fixed by law.

NOTE 1. — A higher rate of interest than that prescribed by law is termed *usury*, and is prohibited by law.

NOTE 2. — When the rate per cent. is not named in notes, or other business documents, the *legal rate* must be taken.

NOTE 3. — In most of the States, and on debts due in the United States, 6 per cent. is the legal rate, although a higher rate may be agreed upon by special contract.

When no rate is mentioned, the legal rate is understood. Interest may be simple or compound. In simple interest the principal alone draws interest. Interest on interest remaining unpaid is considered illegal. In compound interest the entire amount due at regular intervals is converted into a new principal. It is compounded annually, semi-annually or quarterly, and sometimes monthly, according to agreement.

To be an expert in computing interest it is necessary to be familiar with all the methods, and apply the one that will give the correct result with the least labor. To compute interest for years, multiply the principal by the per cent., and you have the interest for one year.

EXAMPLE 1. — What is the interest on $84 at 6 per cent. for 3 years?

$$\$84 \times .06 = \$5.04 \times 3 = \$15.12.$$

IF THE TIME CONSISTS OF MONTHS.

Multiply the principal by the per cent, and you have the interest for 1 year. 1 month being $\frac{1}{12}$ of a year, divide by 12 for the interest of 1 month; multiply this result by the number of months for the interest.

EXAMPLE 2.—What is the interest on $120, at 8 per cent for 8 months?

COMMON METHOD.

$$\begin{array}{r} 120 \\ .08 \\ \hline 12\,)\,9.60 \\ \hline .80 \\ 8 \\ \hline 6.40 \end{array}$$

CANCELLATION.

$$\begin{array}{r|l} & \overset{10}{\cancel{120}} \\ \cancel{12} & 8 \times 8 \times 10 = 6.40 \\ & 8 \end{array}$$

NOTE.—If the time consists of years and months, reduce the years to months, adding the number of months, and proceed as above.

IF THE TIME CONSISTS OF YEARS, MONTHS AND DAYS.

Reduce years to months, adding the number of months, then place $\frac{1}{3}$ of the number of days to the right of the months and proceed as before.

REMARK.—Placing $\frac{1}{3}$ of the days to the right, is reducing the days to the decimal of a month. The reason of this is obvious from the fact that we calculate 30 days for 1 month, then 1 day is $\frac{1}{30}$ of a month, and 3 days $\frac{3}{30}$, or $\frac{1}{10}$, or in decimal form .1; hence by taking $\frac{1}{3}$ of the number of days we obtain tenths of a month.

EXAMPLE 3.—What is the interest on $150, at 9 per cent, for 1 year, 4 months, and 12 days:

COMMON METHOD.

$$\begin{array}{r} 150 \\ .09 \\ \hline 12\,)\,13.50 \\ \hline 1.125 \\ 16.4 \\ \hline 18.4500 \end{array} \qquad \begin{array}{rl} 1 \text{ year} = & 12 \text{ months.} \\ & 4 \quad \text{"} \\ 12 \text{ days} = & .4 \quad \text{"} \\ \hline & 16.4 \text{ months.} \end{array}$$

BY CANCELLATION.

$$\begin{array}{r|l} \cancel{12}\ 4 & 150 \qquad 3 \times 15 = 45 \times 41 = 18.45 \\ & \cancel{9}\ 3 \\ & \cancel{16.4}\ 41 \qquad 41 \times 3 \times 15 = 18.45 \end{array}$$

This operation comes under the head of contractions, and can be multiplied mentally.

ON ALL NOTES THAT BEAR $12 PER ANNUM, OR ANY ALIQUOT PART OR MULTIPLE OF 12.

Any principal that bears $12 per year will bring $1 per month; hence, the time in months must be the interest.

ILLUSTRATION.—If the interest for 1 month is $1, for 15.3 months it is 15.3 times $1, or $15.30; since the multiplication by the figure 1 is altogether superfluous we can dispense with it, and at once say $15.3 or $15.30. Hence the propriety of the following:

RULE.—Reduce years to months, add in the given months, place $\frac{1}{3}$ the number of days to the right, and we have the interest in dimes.

EXAMPLE 1.—Required the interest for $150, at 8 per cent, for 1 year, 6 months, and 11 days.

$$\begin{array}{r} 150 \\ .08 \\ \hline 12.00 \end{array} \qquad \begin{array}{l} \tfrac{1}{3} \text{ of } 11 \text{ days} = 3\tfrac{2}{3}. \\ \begin{array}{ccc} \text{Years.} & \text{Months.} & \text{Days.} \\ 1 & 6 & 11 = 18.3\tfrac{2}{3}, \end{array} \end{array}$$

therefore 18.3\frac{2}{3}$ dimes, or $18.36. Ans.

A note that bears an aliquot part or multiple of 12; such as 6, 18, 24, 36, etc.

EXAMPLE 2.—What is the interest on $300, at 6 per cent, for 3 years, 6 months, and 18 days?

$300		$\frac{1}{3}$ of 18 days = 6.		
.06		Years.	Months.	Days.
18.00		3	6	18 = 42.6.

$42.60 Interest at $12 a year.
21.30 " " $6, or $\frac{1}{2}$ of $12, a year.
63.90

CANCELLATION.

$$\begin{array}{r|l} \overset{2}{\cancel{12}} & \begin{array}{l} 300 \\ \cancel{42.6} \\ \cancel{6} \end{array} \end{array} \qquad 213 \times 300 = 63.90$$

NOTE.—To find the time any sum will double itself at simple interest, simply divide 100 by the rate per cent.

ANOTHER METHOD TO COMPUTE INTEREST FOR DAYS.

RULE.—Find the interest for 1 year and divide by 360 (the number of interest days in a year), and multiply by the number of days for the interest.

EXAMPLE.—Required the interest on $720, for 60 days, at 8 per cent.

```
        $720
         .08
360 ) 57.60 ( .16
       360      60
       2160   9.60
       2160
```

CANCELLATION.

$$\begin{array}{r|l} \cancel{360} & \begin{array}{l} \cancel{720}\ 2 \\ 60 \\ 8 \times 60 \times 2 = 9.60 \end{array} \end{array}$$

REMARK.—In using 360 for a divisor the cipher may be rejected, because it avails nothing in dividing, and makes the divisor ten times as short; your answer will be mills instead of cents, as before; cut off the right hand figure, and you have the interest in cents.

For 10 per cent add $\frac{2}{3}$.
" 12 " multiply by 2.

Example.—What is the interest of $124, at 7 per cent, for 54 days?

$\frac{1}{6}$ of 54 = 9.

$$\begin{array}{r} 124 \\ 9 \\ \hline 1.116 \\ .186 = \frac{1}{6}. \\ \hline \$1.302 \end{array}$$

$$\begin{array}{r|l} 4 & \quad 31 \\ \cancel{36} & \cancel{124} \\ & \cancel{54} \quad 6 \\ \end{array}$$

$7 \times 6 \times 31 = 1.302.$

Bankers' Method.

Banking business being nearly all transacted on the basis of 30, 60, and 90 days, the work can be very much abbreviated and the interest, sometimes, obtained without any calculation whatever.

The following example will best illustrate this rule.

Example.—What is the interest on $120, for 60 days, at 6 per cent?

$$\begin{array}{r|l} \cancel{6} & 120 \\ \cancel{36} & \cancel{60} \quad 10 \times 120 = 1.200 \quad \text{Ans.} \\ & \cancel{6} \end{array}$$

Observe that in this case we cancel the factors in the time and rate, and that the figures in the principal remain unchanged, therefore:

Rule.—For any note at 6 per cent for 60 days remove the decimal point two places to the left, and you have the interest.

Example.—Required the interest on $350, at 6 per cent, for 60 days.

Solution.—Remove the point in the $350 two places to the left, thus: 3.50, and you have the result.

This is the shortest and best rule for days that can be adopted, because it may be applied in any per cent and any number of days. The great superiority of this rule consists in its simplicity, and when once understood is not readily forgotten.

Some accountants have a different rule for every per cent, many of which are not only apt to be forgotten, except by those who apply them daily, but are actually deduced from the above rule.

In reckoning 360 days instead of 365 gives $\frac{5}{365}$, or $\frac{1}{73}$, too much. But the difference is so small that in ordinary transactions it is not noticed. It is now universally adopted in all business transactions.

To find the accurate interest divide by 365 instead of 360.

When the time is less than 1 month, the cents in the principal may be disregarded, because the interest on *that* sum for *that* time would not amount to a cent; when less than 2 months, all under 50 cents, when less than 3 months, all under 33, and so on.

To illustrate we will give the table of divisors for the different per cents. Any sum multiplied by the time in days, and divided by the number opposite the per cent, will give the interest at that per cent.

At 5 %	divide	by	72	At 12 %	divide	by	30
" 6 %	"	"	60	" 15 %	"	"	24
" 7 %	"	"	52	" 20 %	"	"	18
" 8 %	"	"	45	" 24 %	"	"	15
" 9 %	"	"	40	" 40 %	"	"	09
" 10 %	"	"	36				

It will be observed that these divisors are obtained by dividing 360 by the rate per cent; and the student will have to retain in his memory a different divisor for every per cent when, by using 36, once for always

he need remember but one. The great advantage of using 36 must at once be admitted. Some authors use 12 and 30 which, of course, is the same thing; we will now give a few solutions of problems as solved by the old method, and also by cancellation, that the student may perfectly understand them.

EXAMPLE 1.—What is the interest of $80 for 1 year, 5 months, and 12 days?

1 yr., 5 mo., 12 days = 522 days.

```
        80
       .06
      ----
       480
       522
      ----
        960
       960
      2400
      -----
360 ) 250560 ( $6.96
      2160
      ----
       3456
       3240
       ----
        2160
        2160
```

$$\begin{array}{r|l} & 80 \\ \cancel{36} & \cancel{6} \\ \cancel{6} & \cancel{522} \\ & 87 \times 80 = 6.960. \end{array}$$

The student will here notice the vast amount of labor saved in the cancelling method.

EXAMPLE 2.—What is the interest on $48 for 2 years, 3 months, and 6 days, at 8 per cent?

2 years, 3 months, 6 days = 816 days.

$$\begin{array}{r|l} \cancel{6} & \cancel{48}\ \ 8 \\ \cancel{36} & 8 \\ & \cancel{816}\ \ 136 \times 8 \times 8 = 8.704 \end{array}$$

Another short rule for computing interest is called

THE SIX PER CENT BASIS.

TO FIND THE INTEREST FOR MONTHS AT 6 PER CENT.

RULE.—Multiply the principal by one half the number of months; when the principal is dollars only, point off two

places for cents in the product; when dollars and cent point off four places.

Example 1.—What is the interest on $153, at 6 per cent, for eight months?

$\frac{1}{2}$ of 8 = 4

153	
4	
$6.12	Ans.

2	153 × 4 = $6.12 Ans.
~~12~~	~~6~~
	~~8~~ 4

Solving the above by cancellation will show why half the number of months, at 6 per cent, will bring the interest.

TO FIND THE INTEREST FOR DAYS AT 6 PER CENT.

Rule.—Multiply by $\frac{1}{6}$ of the number of days, and the product will be the interest in mills.

Example.—What is the interest on $124, at 6 per cent, for 54 days?

$\frac{1}{6}$ of 54 = 9.

124	
9	
$1.116.	Ans.

~~6~~	124 × 9 = 1.116.
~~36~~	~~54~~ 9
	~~6~~

It will be observed, also, in this that it is an abbreviation of cancellation.

TO FIND THE INTEREST AT ANY GIVEN RATE.

Rule.—Find the interest at 6 per cent as above; divide by 6 for 1 per cent, then multiply by the given rate; or, increase or diminish the result obtained by the rule for 6 per cent, in the same ratio that the rate is increased or diminished.

For	4	per cent	subtract	$\frac{1}{3}$.
"	5	"	"	$\frac{1}{6}$.
"	7	"	add	$\frac{1}{6}$.
"	8	"	"	$\frac{1}{3}$.
"	9	"	"	$\frac{1}{2}$

When the Time is more or less than 60 Days.

Increase or diminish in the same ratio as the time is increased or diminished.

For	90	days	add ½ itself.
"	120	"	multiply by 2.
"	30	"	divide by 2.
"	15	"	" 4.
"	45	"	subtract ¼.
"	20	"	divide by 3.
"	10	"	" 6.
"	6	"	" 10.
"	3	"	" 20.

NOTE.—Nearly all business paper is calculated on 30, 60, 90 days, or an aliquot part or multiple of a month.

EXAMPLE 1.—What is the interest on $120. for 90 days, at 6 per cent?

$1.20 interest for 60 days.
.60 " " ½ of 60 or 30 days.
$1.80 interest for 90 days.

EXAMPLE 2.—What is the interest on $134.24, for 75 days, at 6 per cent?

$1.3424 interest for 60 days.
.3356 " " 15 " or ¼ of 60 days.
$1.6780

The interest at any other rate can be obtained as in preceding rule, or by the following, which will show in what time, at the different rates, any number of dollars will give the interest in cents corresponding with the same figures in the principal. Thus the interest on $140 for 90 days, at 4 per cent, is $1.40.

This rule is no shorter than the other, unless the time corresponds with the same figures in the table, or when it is an aliquot part of the time in the table.

RULE.—When the time and rate correspond with the time and rate in the table, remove the decimal point two places to the left, as in the preceding rule.

4 per cent	for	90	8 per cent	for	45
5 "	"	72	9 "	"	40
6 "	"	60	10 "	"	36
7 "	"	52	12 "	"	30

The cancelling system is very much preferred to this, because it very frequently takes advantage of both time and principal, as will be seen in the following solution:

EXAMPLE.—Required the interest on $540, for 49 days, at 6 per cent?

BANKER'S METHOD.

	$5.40 =	Interest for 60 days.
2)	5.40	
2)	2.70 =	Interest for 30 days.
5)	1.35 =	" " 15 "
3)	.27 =	" " 3 "
	.09 =	" " 1 day.
	$4.41	

CANCELLATION.

$\cancel{6}$	$\cancel{540}$ 90
$\cancel{36}$	49 × 90 = 4410
	$\cancel{6}$

N. B.—Where the time is not an aliquot part of the time in the table, but the principal is, reverse the operation and point off two places in your time for the interest, thus: a note that bears $1.17 in 60 days, at 6 per cent, on $117, is the same as $60 for 117 days.

NOTE.—We have now conclusively proven that *all* the abbreviated processes of computing interest are based entirely upon the system of cancellation.

Indeed, there are many other methods that might abbreviate the work, *if you have examples to suit.*

But the canceling system will give you the advantage at $4\frac{1}{2}$, 6, 8, 9, 12, and 15 per cents, and very frequently of the time and principal. And, to say the least, if the numbers are *all* prime, and you can cancel *none*, you have stated your problem in its simplest form to be solved by any other rule. We therefore recommend without hesitation the adoption of the system of Cancellation, as a *general* and *universal* rule. The student, by close observation and considerable practice, may deduce rules from *this*.

How to find the PRINCIPAL, the rate, time and interest being given.

RULE.—Divide the given interest by the interest on one dollar for your time and rate.

To find the RATE, when principal, time and interest are given.

RULE.—Divide the given interest by the interest on the principal at one per cent.

To find the TIME, principal, rate and interest being given.

RULE.—Divide the given interest by the interest on the principal for one day, the quotient will be the required time in days.

How to Compute Time.

RULE. — Subtract as in compound numbers, reckoning 30 days to the month.

EXAMPLE. — What is the time from January 30th, 1869, to March 13th, 1870?

1870	3	13
1869	1	30
1	1	13
Year.	Month.	Days.

SUGGESTION.—When you are obliged to borrow from the next higher number, in subtraction of compound numbers, subtract the number in the subtrahend from the borrowed number first, then add the number to the minuend, thus, as in above,

$$30 \text{ from } 30 = 0 + 13 = 13.$$

Partial Payments.

The manner of computing interest where partial payments have been made, has given rise to much litigation. The law in the different states on the subject very often does not clearly indicate the principle applicable in all cases. The aim of the law, of course, is to avoid usury and compound interest. The difficulty is in deciding whether the payment shall be applied to liquidate the interest or the principal. The U. S. rule involves compound interest, as often as a payment is made greater than the interest then due. When more than a year intervenes, the U. S. rule is more favorable.

The Vermont rule is more favorable, for there is no compound interest. All payments draw interest.

We give an illustration of a problem under these rules, and the pupil can see the difference readily.

The United States Rule.

I.—The rule for casting interest when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due.

II.—If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due.

III.—If the payment be less than the interest, the surplus of the interest must not be taken to augment the principal; but the interest continues on the former principal until the period when the payments exceed the interest due, and *then* the surplus is to be applied towards discharging the principal, and the interest is to be computed on the balance, as aforesaid.—The above is the decision of Chancellor Kent; Johnson's Chancery Reports, Vol. 1, Page 17, and is adopted by the Supreme Court of the United States.

EXAMPLE 1. PHILADELPHIA, May 1, 1842.

For value received, I promise to pay to the order of J. THORNTON the sum of Three Hundred Dollars, with interest. THOS. CLARK.

The following endorsements were made on this note:

1842, Oct. 16, - - -	$ 60.00
1843, March 4, - - -	17.50
1843, Aug. 27, - - -	28.40
1844, April 1, - - -	132.25.

What was the balance due Sept. 19, 1844?

Amount of the Note......................	$300.00
Interest to Oct. 16, 1842	8.25
First Amount	$308.25
First Payment.........................	60.00
New Principal.........................	$248.25
Interest from Oct. 16, 1842 to Mch. 21, 1843	6 41
Second Amount	$254.66

Second Amount	$254.66
Second Payment	17.50
New Principal	$237.16
Interest from Mch. 21, 1843, to Aug. 27, 1843	6.17
Third Amount	$243.33
Third Payment	28.40
New Principal	$214.93
Interest from Aug. 27, 1843, to Apr. 1, 1844	7.66
New Principal	$222.59
Fourth Payment	132.25
New Principal	$ 90.34
Interest from Apr. 1, 1844, to Sept. 19, 1844	2.52
Balance due Sept. 19, 1844	$ 92.86

The following is called

The Vermont Rule,

And is generally applied when the time is less than a year.

I.—Compute the interest on the whole debt from the time it was due until it is paid.

II.—Compute the interest on all payments, from the time of payment until the time of settlement.

III.—Subtract the amount of all the payments, interest included, from the amount of debt, interest included; the balance will be the amount due.

EXAMPLE 2.

One year after date I promise to pay, to the order of D. B. JONES, the sum of Three Hundred and Sixty Dollars, for value received *with use.*

SHELBY, O., Jan. 1, 1869. JOHN MILES.

N. B.—When no rate is mentioned, 6 per cent. is understood.

WEIGHTS AND MEASURES.

AVOIRDUPOIS WEIGHT.

16 drams (dr.) . .	equal	1 ounce, ...	oz.
16 ounces	"	1 pound, . . .	lb.
25 pounds	"	1 quarter, . . .	qr.
4 quarters . . .	"	1 hundred weight,	cwt.
20 hundred weight	"	1 ton, ...	T.

The term Avoirdupois is derived from the French *avoir du poids*, signifying "to have weight." The pound consists of 7000 Troy grains. This weight is used for weighing almost all articles except gold, silver, platina, and precious stones, which are weighed by Troy Weight.

LONG TON WEIGHT.

28 lbs.	1 quarter.
4 quarters, or 112 lbs.	1 hundred weight.
20 cwt., or 2240 lbs.	1 ton.

This measurement is nearly obsolete. It is allowed at the Custom House in estimating duties, and in the wholesale Coal and Iron trade.

MISCELLANEOUS WEIGHTS.

14	lbs.	Hay	1 Stone.
100	"	Grain	1 Cental.
100	"	Raisins	1 Cask.
100	"	Dry Fish	1 Quintal.
100	"	Nails	1 Keg.
196	"	Flour	1 Barrel.
200	"	Pork	1 Barrel.
240	"	Lime	1 Cask.
280	"	Salt	1 Barrel.

THE STONE WEIGHT

So often spoken of in English measures, is 14 lbs. when weighing wool, feathers, hay, etc.; but a stone of beef, fish, butter, cheese, etc., is only 8 pounds.

HAY.

In England, a truss, when new, is 60 lbs., or 56 lbs. of old hay. A truss of straw, 40 lbs. A load of hay is 36 trusses.

In this country, a load is just what it may happen to weigh; and a ton of hay is either 2,000 lbs. or 2,240 lbs., according to the custom of the locality. A bale of hay is generally considered about 300 lbs., but there is no regularity in the weight.

There is no accurate mode of measuring hay but by weighing it. This, on account of its bulk and character, is very difficult, unless it is baled or otherwise compacted. This difficulty has led farmers to estimate the weight by the bulk or cubic contents, a mode which, from the nature of the commodity, is only approximately correct. Some kinds of hay are light, while others are heavy, their equal bulks varying in weight. But for all ordinary farming purposes of estimating the amount of hay in meadows, mows, and stacks, the following rules will be found sufficient:

As nearly as can be ascertained, 25 cubic yards of average meadow hay, in windrows, make a ton.

When well settled in mows or stacks, 15 or 18 cubic yards make a ton.

When taken out of mows or old stacks, and loaded on wagons, 20 or 25 cubic yards make a ton.

Twenty or twenty-five cubic yards of clover, when dry, make a ton.

TO FIND THE NUMBER OF TONS OF MEADOW HAY RAKED INTO WINDROWS.

RULE. — Multiply the length of the windrow in yards by the width in yards, and that product by the height in yards, and divide by 25; the quotient will be the number of tons in the windrow.

EXAMPLE. — How many tons of hay in a windrow 40 yards long by 2 wide and 2 high?

SOLUTION.—$40 \times 2 \times 2 = 160 \div 25 = 6\frac{2}{5}$. Ans.

TO FIND THE NUMBER OF TONS OF HAY IN A MOW.

RULE. — Multiply the length in yards by the height in yards, and that by the width in yards, and divide the product by 15; the quotient will be the number of tons.

EXAMPLE.—How many tons of well-settled hay in a mow 10 yards long by 6 wide and 8 high?

SOLUTION.—$10 \times 6 \times 8 = 480 \div 15 = 32$ tons.

TO FIND THE NUMBER OF TONS OF HAY IN OLD STACKS.

RULE.—Find the area of the base in square yards, in the table of areas of circles; then multiply the area of the base by half the altitude of the stack in yards, and divide the product by 15; the quotient will be the number of tons.

EXAMPLE. — How many tons of hay in a circular stack, whose diameter at the base is 8 yards, and height 9 yards.

SOLUTION.—50.265, area of base in sq. yards, $\times 4\frac{1}{2}$, half the altitude, $= 226.192 \div 15 = 15.079$ tons.

TO FIND THE NUMBER OF TONS IN LONG SQUARE STACKS.

RULE. — Multiply the length in yards by the width in yards, and that by half the altitude in yards, and divide the product by 15; the quotient will be the number of tons.

EXAMPLE. — How many tons of hay in a square stack 10 yards long, 5 wide, and 9 high?

SOLUTION.—$10 \times 5 \times 4\frac{1}{2} = 225 \div 15 = 15$ tons.

TO FIND THE NUMBER OF TONS OF HAY WHEN TAKEN OUT OF MOWS OR OLD STACKS.

RULE. — Multiply the length of the load in yards by the width in yards, and that by the height in yards, and divide the product by 20; the quotient will be the number of tons.

EXAMPLE.—How many tons of hay can be taken from an old stack, in a load 6 yards long by 3 wide and 3 high?

SOLUTION.—$6 \times 3 \times 3 = 54 \div 20 = 2\frac{7}{10}$ tons.

These estimates are for medium sized mows or stacks. If the hay is piled to a great height, as it often is where horse hay-forks are used, the mow will be much heavier per cubic yard.

AN EASY MODE OF ASCERTAINING THE VALUE OF A GIVEN NUMBER OF LBS. OF HAY, AT A GIVEN PRICE PER TON OF 2000 LBS.

RULE.—Multiply the number of pounds of hay (coal, or anything else which is bought and sold by the ton) by one half the price per ton, pointing off three figures from the right hand; the remaining figures will be the price of the hay (or any article by the ton).

EXAMPLE. — What will be the cost of 658 lbs. of hay, at $7.50 per ton?

SOLUTION. — $7.50 divided by 2 equals $3.75, by which multiply the number of pounds, thus:

$$\begin{array}{r} 658 \\ \$3.75 \\ \hline \$2.46\|750. \end{array} \quad \text{Ans.}$$

NOTE.—3.75 is $\frac{3}{8}$; therefore $\frac{1}{8}$ of $658 = 82$, and $\frac{3}{8}$ is 3 times 82 or $2.46.

A BALE OF COTTON

In Egypt is 90 lbs.; in America a commercial bale is 400 lbs.; though put up to vary from 280 to 720, in different localities.

A bale or bag of Sea Island cotton is 300 lbs.

WOOL.

In England wool is sold by the sack or boll, of 22 stones, which, at 14 lbs. the stone, is 308 lbs.

A pack of wool is 17 stones and 2 lbs., which is rated as a pack load for a horse. It is 240 lbs. A tod of wool is 2 stones of 14 lbs. A wey of wool is 6¼ tods. Two weys, a sack. A clove of wool is half a stone.

A quarter of corn or other grain sold by the bushel is eight imperial bushels, or quarter of a ton.

A ton of liquid measure is 252 gallons.

BUTTER

Is sold by avoirdupois weight, which compares with troy weight as 144 to 175; the troy pound being that much lighter. But 175 troy ounces equal 192 of avoirdupois.

A firkin of butter is 56 lbs.; a tub of butter is 84 lbs.

THE KILOGRAMME OF FRANCE

Is 1000 grammes, and equal to 2 lbs. 2 ozs. 4 grs. avoirdupois.

COAL.

A cubic foot of anthracite coal, before it is prepared for domestic use, will on an average weigh about 93 lbs. When broken for the market it will average about 54 lbs.

TO ESTIMATE THE WEIGHT OF COAL IN ANY GIVEN SPACE.

Rule. — Multiply the contents in cubic feet by 54, for anthracite, or by 50, for bituminous coal, and the product will be the weight in pounds.

1. How many tons of anthracite coal, of 2240 lbs. each, can be stored in a bin 28 ft. long, 20 ft. wide, and 4 ft. deep?

Analysis. — $28 \times 20 \times 4 \times 54 \div 2240 = 54$ tons.

2. How many pounds of bituminous coal in a car 30 ft. long and 7 ft. wide, the depth of the coal being 16 in.?

3 How many pounds of anthracite coal can be placed in a cart which measures 6 ft. in length, $4\frac{1}{2}$ ft. in width, and 16 in. in depth?

4. I wish to build a bin in my cellar to hold 8 tons of anthracite coal, 2240 lbs. to the ton; I have made the length 12 ft., and the width 10 ft.; what must be the height of the bin?

5. How many pounds of bituminous coal can be stored in a space $50 \times 50 \times 12\frac{1}{2}$ ft.?

6. How many tons of anthracite coal, 2000 lbs. to the ton, can be stored in a yard which measures 60 ft. in length, and 30 ft. in width, the depth of the coal being 6 ft.?

7. A dealer purchases 1500 tons of anthracite coal, 2240 lbs. to the ton, which he wishes to store in an inclosure 100 ft. long, and 80 ft. wide; what will be the depth of the coal?

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL.

The following Table shows the weight of a bushel as prescribed by statute, in the several States named.

COMMODITIES.	*California.*	*Connecticut.*	*Illinois.*	*Indiana.*	*Iowa.*	*Kentucky.*	*Louisiana.*	*Maine.*	*Massachusetts.*	*Michigan.*	*Minnesota.*	*Missouri.*	*N. Carolina.*	*N. Hampshire.*	*New Jersey.*	*New York.*	*Ohio.*	*Oregon.*	*Pennsylvania.*	*Rhode Island.*	*Vermont.*	*Wisconsin.*	*Wash. Territ'y.*
Barley,	50	..	48	48	48	48	32	..	46	48	48	48	..	48	48	48	48	46	47	..	46	48	45
Beans,	..	..	60	60	60	60	..	..	..	..	..	60	..	..	..	62	..	..	..	..	..	..	..
Bituminous Coal,	..	..	80	70	80	80	..	..	..	..	..	80	..	..	..	..	..	..	76	..	..	..	..
Blue Grass Seed,	..	..	14	14	14	14	..	..	..	..	..	14	..	..	..	..	..	..	..	..	..	..	..
Buckwheat,	40	45	40	50	52	52	..	..	46	42	42	52	..	50	50	48	..	42	48	..	46	42	42
Castor Beans,	..	..	46	46	46	..	..	..	..	..	..	46	..	..	..	..	..	..	..	..	..	..	..
Clover Seed,	..	..	60	60	60	60	..	..	..	60	60	60	..	..	64	60	60	60	..	..	..	60	60
Dried Apples,	..	..	24	25	24	..	..	..	..	28	28	24	..	..	..	..	..	28	..	..	..	28	28
Dried Peaches,	..	..	33	33	33	..	..	..	..	28	28	33	..	..	..	..	..	28	..	..	..	28	28
Flax Seed,	..	..	56	56	56	56	..	..	..	..	..	56	..	..	55	55	56	..	..	..	..	56	..
Hemp Seed,	..	..	44	44	44	44	..	..	..	..	..	44	..	..	..	..	..	..	..	..	..	..	..
Indian Corn,	52	56	52	56	56	56	56	..	56	56	56	52	..	54	56	58	56	56	56	..	56	56	56
Indian Corn in ear,	..	..	70	68	68	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Indian Corn Meal,	..	..	48	50	..	50	..	50	50	..	..	..	..	..	..	..	..	..	..	50	..	..	..
Oats,	32	28	32	32	35	33⅓	32	30	30	32	32	35	30	..	30	32	32	34	32	..	32	32	36
Onions,	..	..	57	48	57	57	..	..	52	..	..	57	..	..	..	..	..	..	..	50	..	..	50
Potatoes,	..	60	60	60	60	60	..	60	..	..	..	60	60	..	60	60	..	60	..	60	60	60	60
Rye,	54	56	54	56	56	56	32	..	56	56	56	56	..	..	56	56	56	56	56	..	56	56	56
Rye Meal,	..	..	..	..	..	..	..	50	50	..	..	..	..	..	..	..	..	..	..	50	..	..	..
Salt,	..	..	..	50	50	50	..	..	..	..	..	50	..	..	..	56	..	..	..	..	..	..	..
Timothy Seed,	..	..	45	45	45	45	..	..	..	..	..	45	..	..	..	44	..	..	..	..	..	46	..
Wheat,	60	56	60	60	60	60	60	..	60	60	60	60	..	60	60	60	60	60	60	..	60	60	60
Wheat Bran,	..	..	20	..	20	20	..	..	..	..	..	20	..	..	..	..	..	..	..	..	..	..	..

RAILROAD FREIGHT.

GROSS WEIGHTS.

The articles named are billed at actual weights, if possible, but usually at the weights in the table below when it is not convenient to weigh them.

Ale and Beer	320	lbs. per	bbl.	High wines	350	lbs. per	bbl.
" "	170	"	½ "	Hung'n Grass Seed	45	"	bu.
" "	100	"	¼ "	Lime	200	"	bbl.
Apples, dried	24	"	bu.	Malt	38	"	bu.
" green	56	"	"	Millet	45	"	"
" "	150	"	bbl.	Nails	108	"	keg.
Barley	48	"	bu.	Oats	32	"	bu.
Beans, white	60	"	"	Oil	400	"	bbl.
" castor	46	"	"	Onions	57	"	bu.
Beef	320	"	bbl.	Peaches, dried	33	"	"
Bran	20	"	bu.	Pork	320	"	bbl.
Brooms	40	"	doz.	Potatoes, common	150	"	"
Buckwheat	52	"	bu.	" "	60	"	bu.
Cider	350	"	bbl.	" sweet	55	"	"
Charcoal	22	"	bu.	Rye	56	"	"
Clover Seed	60	"	"	Salt, fine	56	"	"
Corn	56	"	"	" "	300	"	bbl.
" in ear	70	"	"	" coarse	350	"	"
" Meal	48	"	"	" in sacks	200	"	sack.
" "	220	"	bbl.	Timothy Seed	45	"	bu.
Eggs	200	"	"	Turnips	56	"	"
Fish	300	"	"	Vinegar	350	"	bbl.
Flax Seed	56	"	bu.	Wheat	60	"	bu.
Flour	200	"	bbl.	Whiskey	350	"	bbl.
Hemp Seed	44	"	bu.	One *ton* weight is 2000 lbs.			

NOTE.—From 18,000 to 20,000 lbs. is considered a car-load in most places, each car itself also weighing about 20,000 lbs.

To Estimate Grain Crops per Acre.

Frame together four light sticks, measuring exactly a foot square inside, and with this in one hand, walk into the field and select a spot of fair average yield, and lower the frame square over as many heads as it will inclose, and shell out the heads thus inclosed carefully, and weigh the grain. It is fair to presume that the proportion will be the 43,560th part of an acre's produce. To prove it go through the field and make ten or twenty similar calculations, and estimate by the mean of the whole number of results. It will certainly enable a farmer to make a closer calculation of what a field will produce than he can by guessing.

The Metric System of Weights and Measures,

With their Equivalents According to the System in Use.

Measures of Length.

Metric Denominations and Values.		Equivalents in Denominations in use.
Myriameter..... =	10,000 meters......	= 6.2137 miles.
Kilometer ...... =	1,000 meters......	= 0.62137 mile or 3,280 ft. 10 in.
Hectometer..... =	100 meters......	= 328 feet and 1 inch.
Dekameter...... =	10 meters......	= 393.7 inches.
Meter .... =	1 meter.......	= 39.37 inches.
Decimeter ...... =	.1 of a meter	= 3.937 inches.
Centimeter ..... =	.01 of a meter	= 0.3937 inch.
Millimeter...... =	.001 of a meter	= 0.0394 inch.

Measures of Surface.

Hectare .... =	10,000 square meters.........	= 2,471 acres.
Are............. =	100 square meters.........	= 119.6 square yards.
Centare......... =	1 square meter..........	= 1,550 square inches.

Measures of Capacity.

Names.	No. Liters.	Cubic Measure.	Dry Measure.	Wine Measure
Kiloliter.. =	1,000.... =	1 meter... =	1,308 cub. yds.. =	264.17 galls.
Hectoliter. =	100.... =	.1 meter... =	2 bu. 3.35 pks.. =	26.417 galls.
Decaliter. =	10.... =	10 decim... =	9.08 quarts..... =	2.6417 galls.
Liter ..... =	1.... =	1 decim... =	0.908 quart..... =	1.0567 qts.
Deciliter.. =	.1.... =	.1 decim... =	6.1022 cub. in.. =	0.845 gill.
Centiliter. =	.01 ... =	10 centim.. =	0.6102 cub. in.. =	0.338 fl'd oz.
Milliliter . =	.001.... =	1 centim.. =	0.061 cub. in... =	0.27 fluid dr.

Weights.

Names.	No. Grams.	Weight of what quantity of water at maximum density.	Avoirdupois Weight.
Millier or tonneau.. =	1,000,000.... =	1 cubic meter.... =	2204.6 lbs.
Quintal............ =	100,000.... =	1 hectoliter ...... =	220.46 lbs.
Myriagram......... =	10,000.... =	10 liters .... =	22.046 lbs.
Kilogram or Kilo .. =	1,000.... =	1 liter .... =	2.2046 lbs.
Hectogram......... =	100.... =	1 deciliter .... =	3.5274 oz.
Dekagram .... =	10.... =	10 c. centimet...... =	0.3527 oz.
Gram .... =	1.... =	1 c. centimet...... =	15.432 grs.
Decigram .... =	.1 ... =	1 c. centimet...... =	1.5432 grs.
Centigram .... =	.01.... =	10 c. millimet..... =	0.1543 gr.
Milligram .... =	.001.... =	1 c. millimet.. .. =	0.0154 gr.

LIQUID OR WINE MEASURE.

4 gills (gi.)	equal	1 pint, . . .	pt.
2 pints	"	1 quart, . . .	qt.
4 quarts	"	1 gallon, . .	gal.

The standard liquid gallon contains 231 cubic inches. — In the old tables were given $31\frac{1}{2}$ gallons = 1 barrel; 63 gallons = 1 hogshead; hogsheads = 1 pipe; 2 pipes = 1 tun. These are now obsolete, because scarcely any cask holds exactly the same amount of liquid.

Square Cisterns.

RULE.—Multiply the length in inches by the width in inches, and that product by the depth in inches, and divide the whole product by 231.

EXAMPLE.—How many gallons in a cistern 6 feet long, 3 feet wide, and 4 feet deep.

SOLUTION.—72 inches in length $\times$ 36 inches' width $\times$ 48 inches' depth $= 124416 \div 231 = 538.59$ gals.

Approximate Method for Square Cisterns.

RULE.—Divide the solid contents in feet by 4, and you will have the contents in barrels of $31\frac{1}{2}$ gallons each.

Take the preceding example: $6 \times 3 \times 4 \div 4 =$ 18 barrels.

NOTE.—This rule gives only the approximate contents, and is based on the following: There are $31\frac{1}{2}$ gallons in a bbl., and 231 cubic inches in a gallon, which, divided by 1728, the number of cubic inches in a foot, we have about 4, and for ordinary calculation often gives the correct quantity for many practical purposes.

Approximate Method for Round Cisterns.

RULE.—Multiply the depth in feet by the diameter in feet, and that product by $1\frac{1}{2}$, and you have the number of barrels of $31\frac{1}{2}$ gallons each.

EXAMPLE.—How many barrels in a round cistern 8 feet deep by 5 feet in diameter?

SOLUTION.—$8 \times 5 \times 1\frac{1}{2} = 60$ barrels.

Capacity of Round Cisterns or Tanks.

Tabular view of the number of gallons contained in the clear between the brickwork for each ten inches in depth:

DIAMETER.	GALLONS.	DIAMETER.	GALLONS.
2 feet equal	19	8 feet equal	313
2½ " "	30	8½ " "	353
3 " "	44	9 " "	396
3½ " "	60	9½ " "	461
4 " "	78	10 " "	489
4½ " "	97	11 " "	592
5 " "	122	12 " "	705
5½ " "	148	13 " "	827
6 " "	176	14 " "	959
6½ " "	207	15 " "	1101
7 " "	240	20 " "	1958
7½ " "	275	25 " "	3059

Circular Cisterns.

RULE.—Find the area of the circle by our table of multiples; then multiply this by the depth in inches, and divide this product by 231.

EXAMPLE.—A cistern is 8 feet in diameter by 5 feet deep. How many gallons does it contain.

SOLUTION. — Multiply the area by 96 inches, the diameter = 7238.2 X 60 inches, the depth = 43429.20 ÷ 231 = 1880 gallons. Ans.

Another Rule for the Measurement of Cylindrical Cisterns.

Take the length, width and depth in feet; multiply these together, and the product by 1865; cut off four figures on the right, and the result will be the contents in barrels.

EXAMPLE. — Find the contents of a cistern 6 feet in diameter and 9 feet deep. Six feet, the length, multiplied by 6 feet, the breadth, and the product by 9, the depth, gives 324, which multiplied by 1865, and four figures cut off, gives 60 barrels and a decimal.

In this case we consider the diameter as being both length and breadth. The reason of the rule is this: a cylinder one foot in diameter and one foot long would measure 1865 ten-thousandths of a barrel. A cylinder 9 times as long would contain 9 times as much, and 6 times as wide, 6 times as much as that. The number 1865 is easy to remember, as it corresponds with the number of a year.

ALE OR BEER MEASURE.

2 pints (pt.) . . .	equal	1 quart, . . .	qt.
4 quarts	"	1 gallon, . . .	gal.
36 gallons	"	1 barrel, . . .	bbl.
54 gallons	"	1 hogshead, .	hhd.

Ale or Beer Measure is used in measuring ale, beer, etc. The gallon consists of 282 cubic inches.

TROY WEIGHT.

24 grains (gr.) . .	equal	1 pennyweight,	dwt.
20 pennyweights .	"	1 ounce, . . .	oz.
12 ounces	"	1 pound, . . .	lb.

The term Troy is said to be derived from *Troyes*, the name of a town in France, where the weight was first used in Europe. The symbol (oz.) is from the Spanish word *onza*, for ounce, and (lb.) from *libra*, a pound.

APOTHECARIES' WEIGHT.

20 grains (gr.) . .	equal	1 scruple, . . .	℈
3 scruples . . .	"	1 dram, ...	ʒ
8 drams	"	1 ounce, ...	℥
12 ounces	"	1 pound, . . .	℔

This weight is used in mixing and retailing medicines. The pound is the same as the pound Troy.

TIME.

60 seconds (sec.) . .	equal	1 minute, . .	m.
60 minutes	"	1 hour, . . .	hr.
24 hours	"	1 day, . . .	da.
7 days	"	1 week, . . .	wk.
4 weeks	"	1 month, . .	mo.
52 weeks	"	1 year, . . .	yr.
12 calendar months .	"	1 year, . . .	yr.

SIGNS AND ABBREVIATIONS USED BY PHYSICIANS IN PRESCRIPTIONS, MEDICAL BOOKS AND JOURNALS.

℞. *Receipt.* Take.
A., āā, *ana, utriusque.* Of each.
℔ *Libra.* A pound.
℥ *Uncia.* An ounce.
f℥ *Fluiduncia.* A fluid ounce.
ʒ *Drachma.* A drachm.
fʒ *Fluidrachma.* A fluid drachm.
℈ *Scrupulum.* A scruple.
ss. *Semissis.* Half.
iss. *Sesqui.* One and a half.
Abdom. *Abdomen.*
Abs. Febr. *Absente febre.* In the absence of fever.
Ad., or Add. *Adde,* or *Addatur.* Add, or let there be added.
Ad Lib. *Ad libitum.* At pleasure.
Altern. Hor. *Alternis horis.* Every other hour.
Aq. *Aqua.* Water.
Aq. Bull. *Aqua bulliens.* Boiling water.
Aq. Comm. *Aqua communis.* Common water.
Aq. Ferv. *Aqua fervens.* Hot water.
Aq. Font. *Aqua fontis.* Spring water.
B. A. *Balneum arenæ.* A Sand bath.
Bib. *Bibe.* Drink.
Bis Ind. *Bis indies.* Twice daily.
Bol. *Bolus.* A large pill.
Bull. *Bulliat* Let it boil.
B. V. *Balneum vaporis.* A vapor bath.
Cap. *Capiat.* Let him take.
Chart. *Chartula.* A small paper.
Cochl. *Cochleare.* A spoonful.
Col. *Cola.* Strain.
Collyr. *Collyrium.* An eye water.
Comp. *Compositus.* Compound.
C., or Cong. *Congius.* A gallon.
Coq, *Coque.* Boil.
Cort. *Cortex.* Bark.
C. M. *Cras Mane.* To-morrow morning.
C. N. *Cras nocte.* To-morrow night.
Crast. *Crastinus.* For to-morrow.
D. *Detur.* Let it be given.
Decub. *Decubitus.* Lying down.
De D. in D. *De die in diem.* From day to day.
Dig. *Digeratur.* Let it be digested.
Dil. *Dilutus.* Dilute.
Dim. *Dimidius.* One-half.
Div. *Divide.* Divide.
Enem. *Enema.* A clyster.
F. *Fiat.* Let it be made.
F. Pil. *Fiat pilula.* Make into a pill.
Feb. Dur. *Febre durante.* During the fever.
Fl. *Fluidus.* Fluid.
Gr. *Granum.* A grain.
Gt. *Gutta.* A drop.—Gtt. *Guttæ.* Drops.
Guttat. *Guttatim.* By drops.
Hor. Decub. *Hora decubitus.* At bed-time.
H. S. *Hora somni.* At bed-time.
Lb. and Lib. *Libra.* A pound weight.
Liq. Liquor.
M. *Misce.* Mix.
Man. *Minipulus.* A handful.
Mic. Pan. *Mica Panis.* Crumb of bread.
Min. *Minimum.* The sixtieth part of a drachm by measure.
Mist. *Mistura.* A mixture.
Muc *Mucilago.* Mucilage.
O. *Octarius.* A pint.
Ol. *Oleum.* Oil.
Omn. Hor. *Omni hora.* Every hour.
Omn. Man. *Omni Mane.* Every morning.
Omn nocte. Every night.
Oz. *Uncia.* An ounce.
P. Æ. *Partes æquales.* Equal parts.
Pil. *Pilula.* A pill.
P. R. N. *Pro re nata.* As occasion may require.
Pulv. *Pulvis.* A powder.
Q. S. *Quantum sufficit.* As much as is sufficient.
Rad. *Radix.* Root.
Rep. *Repetatur.* Let it be repeated.
S. *Signa.* Write.
S. A. *Secundum artem.* According to art.
Sem. *Semen.* Seed.
Si Non Val. *Si non valeat.* If it does not answer.
Si Op. Sit. *Si opus sit.* If there be need.
Sing. *Singulorum.* Of each.
Solv. *Solve.* Dissolve.
Sp. *Spiritus.* Spirit,
Sum. *Sumat.* Let him take.
Sp. Vin. *Spiritus vini.* Spirit of wine.
Syr. *Syrupus.* Syrup.
Tr. Tinct. *Tinctura.* Tincture.
Vs. *Venæsectio.* Venesection.

DRY MEASURE.

2 pints (pt.) . . .	equal	1 quart,	. . . .	qt.
8 quarts	"	1 peck,	. . . .	pk.
4 pecks	"	1 bushel,	. . .	bu.
36 bushels	"	1 chaldron,	. .	ch.

Dry Measure is used for measuring grain, fruit, coal, etc.

To Measure Corn in the Crib.

When the crib is equilateral.

Rule.—Multiply the length in feet by the breadth in feet, and that again by the height in feet, which last product multiply by .63 (the fractional part of a heaped bushel in a cubic foot), and the result will be the heaped bushels of ears. For the number of bushels of shelled corn, multiply by 42 (two-thirds of .63), instead of .63.

EXAMPLE.— Required the number of bushels of shelled corn contained in a crib of ears, 15 feet long by 5 feet wide, and 10 feet high?

15, length, X 5, width, X 10, height, = 750 cubic ft. Then 750 X .63 = 472.50 heaped bushels of ears. Also 750 X .42 = 315 bushels of shelled corn.

In measuring the height, of course the height of the corn is intended. And there will be found to be a difference in measuring corn in this mode between fall and spring, because it shrinks very much in the winter and spring, and settles down.

When the crib is flared at the sides.

Rule.—Multiply half the sum of the top and bottom widths in *feet* by the perpendicular height in *feet*, and that again by the length in feet, which last product multiply by .63 for heaped bushels of ears, and by .42 for the number of bushels of shelled corn.

LONG MEASURE.

12 inches (in.) . . .	equal	1 foot, ...	ft.
3 feet	"	1 yard, . . .	yd.
$5\frac{1}{2}$ yards	"	1 rod, ...	rd.
40 rods	"	1 furlong, . .	fur.
8 furlongs	"	1 mile.	m.

The yard is the standard unit of length. It is formed by dividing a pendulum, which vibrates seconds in a vacuum, at the level of the sea, at the latitude of London, into 391,393 equal parts and taking 360,000 of these parts. From this unit all other measures and weights are derived.

NOTE.—*Cloth measure* is practically out of use. In measuring goods sold by the yard, the yard is divided into *halves*, *fourths*, *eighths* and *sixteenths*. At United States Custom Houses, in estimating duties, the yard is divided into *tenths* and *hundredths*.

For Measuring Heights and Distances.

3 inches, ... , ...	1 palm.
4 "	1 hand.
9 "	1 span.
$3\frac{3}{10}$ feet, ...	1 pace.

Mariner's Measure.

Table used by Mariners in calculating distances on water, and the speed of vessels.

9 inches, ...	1 span.
8 spans or 6 ft., ...	1 fathom.
120 fathoms, ...	1 cable's length.
$7\frac{1}{3}$ cables, ...	1 mile or knot.
6086 ft. nearly, ...	1 " " "
3 miles, or knots, ...	1 league.

NOTE. — The number of knots of the log line run off in half a minute indicates the number of knots of distance a vessel goes per hour. A nautical mile or knot equals nearly $1\frac{1}{6}$ statute miles.

Surveyor's Long Measure,

For measuring boundaries of land, areas, railroads, canals.

$7\frac{92}{100}$ inches	1 link.
25 links, ...	1 rod.
4 rods, ...	1 chain.
80 chains, ...	1 mile.

10 chains long by 1 broad, or 10 sq. chains, 1 acre.

GUNTER'S CHAIN, which is the unit of measure used by surveyors, is 66 feet long, consisting of 100 links.

Measurements are recorded in chains and hundredths. Latterly a steel measuring tape 100 feet long, with each foot divided into tenths, is used by engineers as a substitute for the cumbersome chain.

NOTE. — By scientific persons and revenue officers the inch is divided into *tenths*, *hundredths*, etc. Among mechanics the inch is divided into *eighths*. The division of the inch into 12 parts, called lines, is not now in use.

A standard English mile, which is the measure we use, is 5280 feet in length, 1760 yards, or 320 rods. A strip one rod wide and one mile long is two acres. By this it is easy to calculate the quantity of land taken up by roads, and also how much is wasted by fences.

TABLE

For Geographical and Astronomical Calculations.

1 Geographic mile, ...	1.15 statute miles.
3 " "	1 league.
60 " " or 69.16 statute miles,	1 degree.
360 Degrees, . . .	Circumference of the earth.

NOTE. — The earth's circumference is 24,855½ miles, nearly. The nautical mile is 6075⅘ feet, or 795⅘ feet longer than the common mile.

Shoemaker's Measure.

3 Barleycorns or sizes, ... 1 inch.

Number one, *children's measure*, is 4⅜ inches, and every additional number calls for an increase of ⅓ of an inch in length. Number one, *adults' measure*, is 8½ inches long, with a gradual increase of ⅓ of an inch for additional numbers, so that, for example, number ten measures 11½ inches. This measure corresponds to the number of the *last*, and not to the length of the *sole*.

Surface or Square Measure,

USED IN ASCERTAINING THE EXTENT OF SURFACES, SUCH AS LAND, BOARDS, PLASTERING, PAVING, ETC.

TABLE.

144 Square Inches (sq. in.)	1 square foot, sq. ft.
9 Square Feet, ...	1 square yard, sq. yd.
30¼ Square Yards, . . .	1 sq. rod or perch, sq. rd.; P.
160 Square Rods, ...	1 acre, A.
640 Acres, ...	1 square mile, sq. mi.

Measure 209 feet on each side, and you have a square acre within an inch.

NOTE. — The following gives the comparative size, in square yards, of acres in different countries:

English acre, 4840 square yards; Scotch, 6150; Irish, 7840; Hamburg, 11,545; Amsterdam, 9722; Dantzic, 6650; France (hectare), 11,960; Prussia (morgen), 3053.

This difference should be borne in mind in reading of the products per acre in different countries. Our land measure is that of England.

ARTIFICERS estimate their work as follows:

By the *square foot;* as in glazing, stone-cutting, etc.

By the *square yard*, or by the *square* of 100 square feet; as in plastering, flooring, roofing, paving, etc.

CUBIC MEASURE.

TABLE.

1728 cubic inches (cu. in.), . .	1 cubic foot, cu. ft.
27 cubic feet, ...	1 cubic yard, cu. yd.
40 cubic ft. of round timber or 50 cubic feet of hewn timber	1 ton or load, T.
16 cubic feet, ...	1 cord foot, cd. ft.
8 cord feet or 128 cubic feet	1 cord of wood, Cd.
24¾ cubic feet, ...	1 perch or stone, or masonry, Pch.

TO FIND THE CUBIC CONTENTS OF ANY SOLID BODY.

RULE. — Multiply the length by the breadth, and that product by the thickness.

NOTES.—1. A load of earth contains a cubic yard, and weighs about 3250 lbs.

2. Railway and transportation companies estimate light freight by the number of cubic feet it occupies; but heavy freight is estimated by weight.

3. A pile of wood 4 feet wide, 4 ft. high, and 8 ft. long, contains 1 *cord;* and a *cord foot* is 1 foot in length of such a pile.

4. A perch of stone or masonry is 16½ feet long, 1½ feet wide, and 1 foot high, and contains 24¾ cubic feet.

5. A brick is usually 8 inches long, 4 inches wide, and 2 inches thick; hence 27 bricks make a cubic foot.

6. Joiners, painters, and masons make no allowance for windows, doors, etc. In some places it is customary to allow for one-half the opening. Masons make no allowance for the corners of the walls of houses or of cellars. The size of a cellar is estimated by the measurement of the outside of the wall.

Ton Weight and Ton Measure.

A ton of hay, or any other coarse bulky article usually sold by that measure, is 20 gross hundreds, that is 2240 lbs. But in many places it has become the custom to count only 2000 lbs. for a ton. In freighting ships 42 cubic feet are allowed to a ton; in the measurement of timber 40 solid feet if round, and 50 if square, make a ton.

MASONRY.

A perch of stone is 24.75 cubic feet; when built in the wall, 22 cubic feet make 1 perch, $2\frac{3}{4}$ cubic feet being allowed for the mortar and filling.

Three pecks of lime and four bushels of sand to a perch of wall.

To find the number of perches of stone in walls.

RULE.—Multiply the length in feet by the height in feet, and that by the thickness in feet, and divide the product by 22, and the quotient will be the number of perches of stone in the wall.

EXAMPLE.—How many perches of stone contained in a wall 40 ft. long, 20 ft. high, and 18 inches thick?

SOLUTION.—40 ft., length, $\times$ 20 ft., height, $\times$ $1\frac{1}{2}$ ft., thickness, $=$ 1200 $\div$ 22 $=$ 54.54 perches. Ans.

NOTE.—To find the number of perches of *masonry*, divide the product, as above, by 24.75, instead of 22.

Brickwork.

The dimensions of common bricks are from $7\frac{3}{4}$ to 8 inches long, by $4\frac{1}{4}$ wide, and $2\frac{1}{2}$ thick. Front bricks are $8\frac{1}{4}$ inches long, by $4\frac{1}{2}$ wide, and $2\frac{1}{2}$ thick.

The usual size of fire bricks is $9\frac{1}{8}$ inches long, by $4\frac{5}{8}$ wide, by $2\frac{3}{8}$ thick.

Twenty common bricks to a cubic foot when laid; 15 common bricks to a foot of 8-inch wall when laid.

To find the number of common bricks in a wall.

RULE. — Multiply the length of the wall in feet by the height in feet, and that by its thickness in feet, and that again by 20, and the product will be the number of bricks in the wall.

EXAMPLE. — How many common bricks in a wall 40 feet long by 20 feet high and 12 inches thick?

SOLUTION.—40 ft., length, X 20 ft., height, X 1 ft., thick, X 20 = 16,000. Ans.

NOTE.—For walls 8 inches thick, multiply the length in feet by the height in feet, and that by 15, and the product will be the number of bricks in the wall.

When the wall is perforated by doors and windows, or other openings, find the sum of their cubic feet by severally multiplying their lengths and widths and thicknesses in feet together, and deducting the whole from the cubic contents of the wall, including the openings, before multiplying by 15 or 20, as above.

Laths.

Laths are $1\frac{1}{4}$ to $1\frac{1}{2}$ inches wide by 4 feet long, are usually set $\frac{1}{4}$ inch apart, and a bundle contains 100.

Short Approximate Method for Estimating Stone Work.

RULE. — Multiply the length in feet by the height in feet by the thickness in feet, and that product by 4, cutting off the two right-hand figures.

EXAMPLE.—How many perches of stone in a wall 40 ft. long, 20 ft. high, and 2 ft. thick?

40 X 20 X 2 X 4 = 64.00, or 64 perches.

CARPENTERS' ESTIMATES.

SHINGLES are usually 16 inches long and, on an average, very nearly 4 inches wide.

They are put up in bundles 20 inches wide and of 24 courses. Four such bundles contain 1000 of shingles.

1000 of shingles laid 4 inches to the weather are estimated to cover 109 square feet; laid $4\frac{1}{2}$ inches to the weather, to cover 120 square feet; and laid 5 inches to the weather, to cover 133 square feet.

For laying 1000 shingles carpenters allow about 6 pounds of 4-penny nails, or 5 pounds [illegible]-penny nails.

NUMBER OF SHINGLES IN A ROOF. **First find the number of square inches in one side of the roof; cut off the right hand or unit figure, and the result will be the number of shingles required to cover both sides of the roof, laying five inches to the weather. The ridge-board provides for the double courses at the bottom. Illustration: Length of roof, 100 feet, width of one side, 30 feet, — $100 \times 30 \times 44 = 432{,}000$. Cutting off the right-hand figure, we have 43,200 as the number of shingles required.**

CLAPBOARDS are usually 4 feet long, and are estimated by the 1000.

100 of clapboards laid 4 inches to the weather are estimated to cover $133\frac{1}{3}$ square feet; laid $4\frac{1}{2}$ inches to the weather, to cover 150 square feet; and laid 5 inches to the weather, to cover $166\frac{2}{3}$ square feet.

For laying 100 of clapboards carpenters allow about $3\frac{1}{2}$ pounds of 5-penny nails.

LATHS are usually 4 feet long, $1\frac{1}{2}$ inches wide, and $\frac{1}{4}$ inch thick, and are put up in bundles containing 100.

1000 of laths, set $\frac{1}{4}$ of an inch apart, are estimated to cover 55 square yards.

For setting 1000 of laths carpenters allow about 7 pounds of 3-penny fine nails.

TABLE OF MULTIPLES.

For the practical convenience of those who have occasion to refer to mensuration, we have arranged the following useful Table of Multiples. It covers the whole ground of practical Geometry.

Diameter of a circle × 3.1416 = Circumference.

Radius of a circle × 6.283185 = Circumference.

Square of the radius of a circle × 3.1416 = Area.

Square of the diameter of a circle × 0.7854 = Area.

Square of the circumference of a circle × 0.07958 = Area.

Half the circumference of a circle × by half its diameter = Area.

Circumference of a circle × 0.159155 = Radius.

Square root of the area of a circle × 0.56419 = Radius.

Circumference of a circle × 0.31831 = Diameter.

Square root of the area of a circle × 1.12839 = Diameter.

Diameter of a circle × 0.86 = Side of inscribed equilateral triangle.

Diameter of a circle × 0.7071 = Side of an inscribed square.

Circumference of a circle × 0.226 = Side of an inscribed Square.

Circumference of a circle × 0.282 = Side of an equal square.

Diameter of a circle × 0.8862 = Side of an equal square.

Base of a triangle × by ½ the altitude = Area.

Multiply both diameters and .7854 together = Area of an ellipse.

Surface of a sphere × by 1-6 of its diameter = Solidity.

Circumference of a sphere × by its diameter = Surface.

Square of the diameter of a sphere × 3.1416 = Surface.

Square of the circumference of a sphere × 0.3183 = Surface.

Cube of the diameter of a sphere × 0.5236 = Solidity.

Cube of the radius of a sphere × 4.1888 = Solidity.

Cube of the circumference of a sphere × 0.016887 = Solidity.

Square root of the surface of a sphere × 0.56419 = Diameter.

Square root of the surface of a sphere × 1.772454 = Circumference.

Cube root of the solidity of a sphere × 1.2407 = Diameter.

Cube root of the solidity of a sphere × 3.8978 = Circumference.

Radius of a sphere × 1.1547 = Side of an inscribed cube.

Sqare root of (1-3 of the square of) the diameter of a sphere = Side of inscribed cube.

Area of its base × by 1-3 of its altitude = Solidity of a cone or pyramid whether round, square, or triangular.

Area of one of its sides × 6 = Surface of a cube.

Altitude of trapezoid × ½ the sum of its parallel sides = Area.

Paper Table for Printers' and Publishers' Use,

Showing the quantity of paper required for printing 1000 *copies (including* 56 *extra copies to allow for wastage), of any usual sized Book from* 8vo. *down to* 32mo. *If the quantity required is not found in the Table, double or treble some suitable number of pages or quantity of paper.*

No. of Forms.	8vo. Pages.	12mo. Pages.	16mo. Pages.	24mo. Pages.	32mo. Pages.	1000 Copies. Rs.	Qs.
1	8	12	16	24	32	1	2
2	16	24	32	48	64	2	4
3	24	36	48	72	96	3	6
4	32	48	64	96	128	4	8
5	40	60	80	120	160	5	10
6	48	72	96	144	192	6	12
7	56	84	112	168	224	7	14
8	64	96	128	192	256	8	16
9	72	108	144	216	288	9	18
10	80	120	160	240	320	11	
11	88	132	176	264	352	12	2
12	96	144	192	288	384	13	4
13	104	156	208	312	416	14	6
14	112	168	224	336	448	15	8
15	120	180	240	360	480	16	10
16	128	192	256	384	512	17	12
17	136	204	272	408		18	14
18	144	216	288	432		19	16
19	152	228	304	456		20	18
20	160	240	320	480		22	
21	168	252	336	504		23	2
22	176	264	352			24	4
23	184	276	368			25	6
24	192	288	384			26	8
25	200	300	400			27	10
26	208	312	416			28	12
27	216	324	432			29	14
28	224	336	448			30	16
29	232	348	464			31	18
30	240	360	480			33	
31	248	372	496			34	2
32	256	384	512			35	4
33	264	396	528			36	6
34	272	408	544			37	8
35	280	420	560			38	10
36	288	432	576			39	12
37	296	444	592			40	14
38	304	456	608			41	16
39	312	468				42	18
40	320	480				44	

Names and Dimensions of Various Sizes of Paper.

PRINT.

Medium........19 x 24	Double Medium.24 x 38
Royal (20 x 24) .20 x 25	Double Royal...26 x 40
Super Royal,....22 x 28	D'ble Super Roy'l28 x 42
Imperial, ...22 x 32	D'ble Super Roy'l29 x 43
Med'm and a half,24 x 30	Broad Twelves ..23 x 41
Small Double Medium ....24 x 36	Double Imperial.32 x 46

FOLDED.

Billet Note ......6 x 8	Letter ....10 x 16
Octavo Note.....7 x 9	Commerc'l Letter 11 x 17
Commercial Note 8 x 10	Packet Post 11½x 18
Packet Note .....9 x 11	Foolscap 12½x 16
Bath Note.......8½x 14	

FLAT.

Legal Cap......13 x 16	Medium.........18 x 23
Flat Cap ....14 x 17	Royal..........19 x 24
Crown ....15 x 19	Super Royal ....20 x 28
D'ble Flat Letter 16 x 20	Imperial ....22 x 30
Demy..........16 x 24	Double Demy...21 x 31
Folio Post......17 x 22	Elephant 22¼x27¾
Check Folio ....17 x 24	Columbier......23 x31¼
Double Cap.....17 x 28	Atlas ....26 x 33
Extra Size Folio.19 x 24	Double Elephant 26 x 40

NAILS.

The different sorts of nails are named either from the use to which they are applied, or from their shape, as shingle, floor, ship-carpenters' and horse-shoe nails; rose-heads, diamonds, &c. The term penny, when used to mark the size of nails, is supposed to be a corruption of pound. Thus a four-penny nail was such that one thousand of them weighed four pounds, ten-penny such that one thousand weighed ten pounds, &c.

RATES.

Although nails are sold at various prices, according to the size. quotations called *Rates* give one price only. meaning that all sizes from 10d to 60d inclusive, sell for the rate named. For prices on other sizes add extra to the price named, according to the following table.

EXTRAS.

2d Fine	$4.00	6d Floor	$1.25
2d	3.00	8d & 9d Floor	1.00
3d Fine	2.25	10d Floor, and larger	.75
3d	1.50	6d Brad Head	1.50
4d Light	1.50	8d "	1.25
4d Swedes, com.	1.50	10d " and larger	1.00
4d " bar	4.00	3d Cooper & Slate	2.00
4d & 5d Common,	.75	4d " "	1.25
6d & 7d Fence and	.50	5d " "	1.25
8d & 9d Sheathing.	.25	6d " "	1.00
Spikes	.25	8d " "	.75
3d Fine Box	4.00	Tin Roofing	3.00
3d Box.	3.00	Steel Nails	1.00
4d "	2.00	1 in. Clinch	4.00
5d "	1.25	$1\frac{1}{4}$ in. "	3.50
6d & 7d Box	1.00	$1\frac{1}{2}$ in. "	3.00
8d & 9d "	.75	$1\frac{3}{4}$ in. "	2.50
10d Box, and larger.	.50	2 & $2\frac{1}{4}$ in Clinch	2.00
4d Fine Finishing	2.25	$2\frac{1}{2}$ & $2\frac{3}{4}$ in. "	1.75
5d "	2.00	3 & $3\frac{1}{2}$ in. "	1.50
6d "	1.75	Half Casks, additional	.25
8d "	1.50	Galvanizing..	2.50
10d "	1.25	Tinning	3.00
1 in. Barrel	2.50		
$1\frac{1}{8}$ in. "	2.00		

LENGTH OF NAILS.

2d	3d	4d	5d	6d	7d	8d	9d	10d	12d	20d	30d	40d	50d	60d
1 in.	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6

NUMBER OF NAILS IN A POUND.

2d	4d	5d	6d	7d	8d	10d	12d	20d	30d	40d	50d
557	353	232	167	141	101	68	54	34	16	12	10

HOW TO MEASURE LAND.

Land can be measured with satisfactory accuracy for many purposes, by pacing. Five paces are equal to one lineal rod. A man having long legs usually measures more than a rod at five paces, while a short legged man will be obliged to step unnaturally long to measure a rod at five paces. The correct way is to measure 16½ feet on level ground, then practice gauging the steps until one can measure one rod at every five steps, then one hundred steps or paces will be equal to twenty rods. If a plat of land be two hundred paces long and fifty paces wide, call every five paces a rod, multiply the rods in length by the rods in width, and divide the product by 160, the square rods in an acre. Thus: 100 paces = 20 rods, and 50 paces = 10 rods; $10 \times 20 = 200$ square rods, which, divided by 160, gives 1¼ acres.

A square acre is about 208 feet 8½ inches on every side.

In order to lay out one acre of land four times as long as the width, the length must be 417 feet 5 inches, and the width 104 feet 4 inches.

Twenty feet front and 2,187 feet deep is one acre.

Twenty-five feet front and 1,742¼ feet deep is one acre.

Thirty-three feet front and 1,320 feet deep is one acre.

Forty feet front and 1,089 feet deep is one acre.

Fifty feet front and 876½ feet deep is one acre.

One hundred feet front and 435½ feet deep is one acre.

In one square acre there are 43,560 superficial feet.

TRANSACTIONS WITH BANKS.

Make your deposits in the bank as *early in the day as you conveniently can*, and never without your bank-book.

For your own security, it is well to have ONE PARTICULAR PERSON to do your business at the bank, who shall be competent to take charge of the money and papers you intrust to his care, and sufficiently intelligent to understand and properly deliver the messages and explanations you may have occasion to make; also, that you write or stamp OVER YOUR INDORSEMENT, upon all checks which you send to be deposited to your credit in the bank, the words "FOR DEPOSIT TO OUR CREDIT," which will prevent their being used for any other purpose.

Always use the deposit tickets furnished by the bank, and examine the date and indorsement of every check. When checks are deposited the banks require them to be indorsed by the depositor, whether drawn to his order or not.

Keep your check-book, when not in use, under your own lock and key. Make it a rule to give checks only out of YOUR OWN CHECK-BOOK.

Draw as few checks as possible. When you have several sums to pay, draw ONE CHECK for the whole, and take notes of such denominations as will enable you to distribute the amount among those you intend it for.

Do not allow your bank-book to run too long without being balanced, and when returned by the bank compare it with your own account, and examine your cancelled checks without delay. If you wish to preserve your cancelled checks, deface or destroy the signature as soon as returned, in a manner that will

prevent their being copied, and place the checks out of the reach of others.

In filling up checks, do not leave space in which the amount may be increased. It has been decided that when a check is so carelessly drawn that an alteration may be easily made, the loss arising from the alteration, if any, must be borne by the drawer.

Write your signature with your usual freedom, and never vary the style of it.

Offer notes for discount or collection in good season. Do not put off the offering of notes for discount until the last day of your need. When notes are discounted or collected for you, hand your bank-book to the clerk, that they may be entered in it to your credit.

BROKERS' TECHNICALITIES.

A Bull is one who operates to depress the value of stocks, that he may buy for a rise.

A Bear is one who sells stocks for future delivery, which he does not own at the time of sale.

A Corner is when the Bears cannot buy or borrow the stock to deliver in fulfillment of their contracts.

Overloaded is when the Bulls cannot take and pay for the stock they have purchased.

Short is when a person or party sells stocks when they have none, and expect to buy or borrow in time to deliver.

Long is when a person or party has a plentiful supply of stocks.

A Pool or Ring is a combination formed to control the price of stocks.

A broker is said to Carry stock for a customer when he has bought and is holding it for his account.

A Wash is a pretended sale by special agreement between buyer and seller, for the purpose of getting a quotation reported.

A Put and Call is when a person gives so much per cent. for the option of buying or selling so much stock on a certain fixed day, at a price fixed the day the option is given.

MARKING GOODS.

In buying goods the merchant is often at a loss to know whether the price of the article suits his market or not; and if he is not a good accountant it often takes him some time to determine. Those who buy largely can best appreciate the value of a short method of calculating the percentage desired.

If you wish to calculate the per cent. on a single article, the following is considered the best method. If you wish to sell an article at any of the following per cents., say the article cost 70 cents, and you wish to make

10%	Divide by 10,	Multiply by 11	=77.
20 "	" 10,	" 12	=84.
25 "	Multiply by 10,	Divide " 8	=87½.
30 "	Divide by 10,	Multiply " 13	=91.
33⅓	Add ⅓ of itself.		=93⅓.
33⅓%	Divide by 3,	Multiply by 4	=93⅓.
50 "	Add ½ of itself.		=1.05.

Another method of marking 25 per cent. profit is to cut off the right-hand figure, and you have the price in shillings and pence: thus, if you buy an article for 60 cents, and wish to gain 25 per cent., cut off the right-hand figure and you have 6 shillings of $12\frac{1}{2}$ cents each, or 75 cents, the cost with 25 per cent added.

If the figure you cut off is not a cipher, add $\frac{1}{4}$, thus: 3 cents, add $\frac{3}{4}$; 5 cents, $1\frac{1}{4}$; 6 cents, $1\frac{1}{2}$, etc.

Example.—Suppose an article cost 74 cents, and you wish to make it 25 per cent. advance, cut off the right-hand figure and you have 7 s. 4 d., $4 = \frac{4}{4} = 1$, added to $4 = 5$, 7 s. $= 87\frac{1}{2} + 5 = 92\frac{1}{2}$ cents.

How to mark an article bought by the dozen, to make 20 per cent.

Remove the decimal point one place to the left.

Example.—Suppose a lot of hats cost \$2.50 per dozen, by removing the decimal point one place to the left we have 20 per cent. and cost, or .25 apiece for the hats. To ascertain any other per cent., we take the basis at 20 per cent. and add or subtract, as the case may be.

To make 25 per cent., remove the point one place to the left, and add $\frac{1}{24}$.

To make	30 %	add	$\frac{1}{12}$	itself.
“	$33\frac{1}{3}$ “	“	$\frac{1}{9}$	“
“	35 “	“	$\frac{1}{8}$	“
“	$38\frac{1}{2}$ “	“	$\frac{1}{3}$	“
“	40 “	“	$\frac{1}{6}$	“
“	44 “	“	$\frac{1}{5}$	“
“	50 “	“	$\frac{1}{4}$	“
“	60 “	“	$\frac{1}{3}$	“
“	80 “	“	$\frac{1}{2}$	“

These additions must be made after removing the point as above directed, and this sum will always be the selling price of a single article.

The above table contains all the per cents. generally used in business, and can easily be applied.

This rule is very valuable to the merchant in buying goods; suppose he buys his goods at auction, he does not have sufficient time to make extensive calculations before the goods are cried off. But by knowing that at 20 per cent. profit, he need not change a figure, he can tell instantly whether he can afford to buy those goods or not.

MARKING GOODS.

It is customary for merchants to have a private mark, denoting the cost and often the selling price. These marks are sometimes made up of peculiar characters, but mostly letters of the alphabet that represent the nine digits. For example:

BLACK HORSE
1 2 3 4 5 6 7 8 9 0

Suppose an article cost $2.25, and you wish to sell it for $5.00, the mark would be thus: $\frac{\text{llk}}{\text{kee}}$

Usually they have what is called a repeater, that is to be used where a letter is repeated, as above suppose G to be the repeater; then instead of using the letters L and E twice, we insert the repeater, thus: $\frac{\text{lgk}}{\text{keg}}$. It sometimes happens that there are but two letters in the cost price and three in the selling price; to avoid this, place the letter representing 0 as the first letter in the cost, thus: write 75 cents cost. 1.00 selling price: $\frac{\text{eok}}{\text{beg}}$.

HOW TO TELL

THE DAY OF THE WEEK,

THE DAY OF THE MONTH,

THE MONTH IN THE YEAR,

THE AGE IN YEARS,

WITHOUT ASKING A SINGLE QUESTION.

PROCESS.

Ask the person you wish to figure out the above facts for, to write down first, the day of the week on which he or she was born; if this is not known, ascertain by preceding method; next, the day of the month, next, the month in the year, then multiply the whole by 2, add 5, multiply by 50, add age, subtract 365, add 115, the result will be; the first figure will be the day of the week; the next, the day of the month; the next the month in the year, and the last the age in years.

EXAMPLE.—I was born Wednesday, May 31, 1843.

PROCESS.—1st. Write 4 as the first figure, because Wednesday is the 4th day of the week.

2d. Write 5 as the second figure, because May is the 5th month.

3d. Write 31 as the 3d and 4th figures, because this is the day on which I was born. The figures therefore read,

```
    4 5 3 1
          2 multiply.
    -------
    9 0 6 2
          5 add.
    -------
    9 0 6 7
        5 0 multiply.
  ---------
4 5 3 3 5 0
        4 1 add age.
-----------
4 5 3 3 9 1
      3 6 5 subtract.
-----------
4 5 3 0 2 6
      1 1 5 add.
-----------
4, 5, 3 1. 4 1
```

Day of the week. Month. Day of the month. Age.

EXAMPLE 2d.—A friend was born April 6th.

Take number of month and day of month.

```
    5 6
      2 multiply.
  -----
  1 1 2
      5 added.
  -----
  1 1 7
    5 0 multiply.
-------
5 8 5 0
    5 0 add age.
-------
5 9 0 0
  3 6 5 subtract.
-------
5 5 3 5
  1 1 5 add.
-------
5, 6, 5 0
```

HOW TO TELL THE DAY OF THE WEEK.

A scientific method of telling immediately what day of the week any date transpired or will transpire, from the commencement of the Christian Era for the term of three thousand years.

MONTHLY TABLE.

The ratio to add for each month will be found in the following table :

Ratio of June is..........0	Ratio of October is.......3
Ratio of September is....1	Ratio of May is...... ...4
Ratio of December is.....1	Ratio of August is........5
Ratio of April is.........2	Ratio of March is........6
Ratio of July is 2	Ratio of February is......6
Ratio of January is.......3	Ratio of November is....6

Note.—On Leap Year the Ratio of January is 2, and the ratio of February is 5. The ratio of the other ten months do not change on Leap Years.

CENTENNIAL TABLE.

The ratio to add for each century will be found in the following table :

Christian Era.	200,	900,	1800,	2200,	2600,	3000,	ratio is.........0
	300,	1000,	....	....	....	....	ratio is.........6
	400,	1100,	1900,	2300,	2700,	....	ratio is.........5
	500,	1200,	1600,	2000,	2400,	2800	ratio is.........4
	600,	1300,	....	....	....	....	ratio is.........3
000,	700.	1400,	1700	2100	2500	2900	ratio is.........2
100,	800,	1500,	....	....	....	....	ratio is.. 1

Note.—The figure opposite each century is its ratio; thus the ratio for 200, 900, etc., is 0. To find the ratio of any century, first find the century in the above table then run the eye along the line until you arrive at the end; the small figure at the end is its ratio.

METHOD OF OPERATION.

Rule.*—To the given year add its fourth part, rejecting the fractions; to this sum add the day of the month; then add the ratio of the month and the ratio of the century. Divide this sum by 7; the remainder is the day of the week, counting Sunday as the first, Monday as the second, Tuesday as the third, Wednesday as the fourth, Thursday as the fifth, Friday as the sixth, Saturday as the seventh; the remainder for Saturday will be 0 or zero.

Example 1.—Required the day of the week for the 4th of July, 1810.

To the given year, which is........................	10
Add its fourth part, rejecting fractions..............	2
Now add the day of the month, which is............	4
Now add the ratio of July, which is..................	2
Now add the ratio of 1800, which is.................	0
Divide the whole sum by 7.	7 | 18—4
	2

We have 4 for a remainder which signifies the fourth day of the week, or Wednesday.

Note.—In finding the day of the week for the present century, no attention need be paid to the *centennial ratio*, as it is 0.

Example 2.—Required the day of the week for the 2d of June, 1805.

To the given year, which is........................	5
Add its fourth part, rejecting fractions.............	1
Now add the day of the month, which is............	2
Now add the ratio of June, which is..................	0
Divide the whole sum by 7.	7 | 8—1
	1

We have 1 for a remainder, which signifies the first day of the week, or Sunday.

The Declaration of American Independence was signed July 4, 1776. Required the day of the week.

* When dividing the year by 4, always leave off the centuries. We divide by 4 to find the number of Leap Years.

To the given year, which is........................76
Add its fourth part, rejecting fractions.... 19
Now add the day of the month, which is 4
Now add the ratio of July, which is................ 2
Now add the ratio of 1700, which is................. 2

Divide the whole sum by 7, 7 | 103—5
14

We have 5 for a remainder, which signifies the fifth day of the week, or Thursday.

The Pilgrim Fathers landed on Plymouth Rock Dec. 20, 1620. Required the day of the week.

To the given year, which is.........................20
Add its fourth part, rejecting fractions 5
Now add the day of the month, which is.............20
Now add the ratio of December, which is 1
Now add the ratio of 1600, which is................. 4

Divide the whole by 7, 7 | 50—1
7

We have 1 for a remainder, which signifies the first day of the week, or Sundav.

AMUSING ARITHMETIC.

Under the head of Amusing Arithmetic we give a collection of problems particularly adapted to the social circle, or the fireside, of a winter evening. The most of those problems are in the form of puzzles, and some of them particularly amusing. The majority of them are very old, their parentage being entirely unknown, so that no credit can be given to their authors. This is believed to be the largest collection ever published.

1. Think of a number of 3 or more figures, divide by 9, and name the remainder; erase one figure of the number, divide by 9, and tell me the remainder, and I will tell you what figure you erased.

METHOD.—If the second remainder is less than the first, the figure erased is the difference between the remainders; but if the second remainder is greater than the first, the figure erased equals 9, minus the difference of the remainders.

2. Think of a number, multiply it by 3, and multiply it also by 4, take the sum of the squares of the products, extract the square root of this sum, divide by the first number, and I will name the quotient.

METHOD.—The quotient will always be 5. The same will be also true if we have them multiply and divide by the same multiples of 3, 4, and 5, as 6, 8, 10, &c. If we have them divide by 5, it will give the number they commenced with.

3. Think of a number, multiply it by 5, also by 12; square each product, take their sum, extract the square root, divide by the number commenced with, and I will name the quotient.

METHOD.—The quotient is always 13. To give variety it is well to use multiples of 5, 12; as 10, 24, &c., and then the quotient is 26, &c.

4. Think of a number composed of two unequal digits, invert the digits, take the difference between this and the original number, name one of the digits and I will name the other.

METHOD.—The sum of the digits in the difference is always 9; hence when one is named, the other equals 9 minus the one named.

5. Take any number consisting of three consecutive digits and permutate them, making 6 numbers, and take the sum of these numbers, divide by 6, and tell me the result, and I will tell you the digits of the number taken.

METHOD.—The quotient consists of three equal digits; the digits of the number taken are, 1st, one of these equal digits; 2d, this digit increased by a unit; 3d, this digit diminished by a unit. The same principle holds when the digits of the number taken differ by 2, 3, or 4. It is a very pretty problem to prove that the sum is always divisible by 9, and 18.

6. Think of a number greater than 3, multiply it by 3; if *even*, divide it by 2; if *odd*, add 1, and then divide by two. Multiply the quotient by 3; if *even*, divide by 2; if *odd*, add 1, and then divide by 2. Now divide by 9 and tell the quotient, without the remainder, and I will tell you the number thought of.

METHOD.—If *even* both times, multiply the quotient by 4; if *even* 2d, and *odd* 1st, multiply by 4, and add 1; if even 1st, and odd 2d, multiply by 4, and add 2; if odd both times, multiply by 4, and add 3.

7. Take any number, divide it by 9 and name the remainder. Multiply the number by some number which I name, and divide this product by 9, and I will name the remainder.

METHOD.—To tell the remainder, I multiply the first remainder by the number by which I told them to multiply the given number, and divide this product by 9. The remainder is the second number that they obtained.

8. A and B have an 8 gallon cask full of wine, which they wish to divide into two equal parts, and the only measures they have are a 5 gallon cask and a 3 gallon cask. How shall they make the division with these two vessels?

METHOD.—Fill the 3 and pour it into the 5, then fill it again, and from it fill up the 5, which will leave one gallon in the 3 gallon keg; empty the 5 in to the 8, and pour the one from the 3 into the 5; fill the 3 again and empty into the 5; then there are four gallons in the 5 gallon keg, and the same left in the 8.

9. Two men have 24 ounces of fluid, which they wish to divide between them equally. How shall they effect the division, provided they have only three vessels, one containing 5 ozs., the other 11 ozs., and the third 13 ozs.

10. Two men, stopping at an oyster saloon, laid a wager as to which could eat the most oysters. One eat ninety-nine, and the other eat a hundred and won. How many did both eat?

REMARK.—The "catch" is in "a hundred and *won*." When this is repeated it sounds as if it meant "one eat 99 and the other eat 101;" hence the result usually given is 200. The correct result, of course, is 199.

11. Six ears of corn are in a hollow stump. How long will it take a squirrel to carry them all out, if he takes out three ears a day?

REMARK.—The "catch" is in the word *ears*. He carries out two ears on his head, and one ear of corn each day; hence it will take him 6 days.

12. A and B went to market with 30 pigs each. A sold his at 2 for $1, and B at the rate of 3 for $1, and they, together, received $25. The next day A went to market alone with sixty pigs, and, wishing

to sell at the same rate, sold them at 5 for $2, and received only $24. Why should he not receive as much as when B owned half of the pigs?

METHOD—The insinuation that the first lot were sold at the rate of 5 for $2. being only true in part. They commence selling at that rate, but after making 10 sales, A's pigs are exhausted, and they have received $20.; B still has 10, which he sells at "2 for a dollar," and of course receives $5; whereas, had he sold them at the rate of 5 for $2, he would have received but $4.

13. In the bottom of a well, 45 feet deep, there was a frog which commenced travelling towards the top. In his journey he ascended 3 feet every day, but fell back two feet every night. In how many days did he get out of the well?

14. A man having a fox, a goose, and some corn, came to a river which it was necessary to cross. He could, however, take only *one* across at a time, and if he left the goose and corn, while he took the fox over, the goose would eat the corn; but if he left the fox and goose, the fox would kill the goose. How shall he get them all safely over?

METHOD.—Let him first take over the goose, leaving the fox and the corn, then let him take over the fox, and bring back the goose, then take over the corn, and lastly, take over the goose again.

15. A man went to a store and purchased a pair of boots worth $5, and hands out a $50 bill to pay for them; the merchant, not being able to make the change, passes over the street to a broker and gets the bill changed, and then returns and gives the man who bought the boots his change. After the purchaser of the boots has been gone a few hours, the broker, finding the bill to be a counterfeit, returns and demands $50 of good money from the

merchant. How much did the merchant lose by the operation.

REMARK.—At first glance some say \$45 and the boots; some, \$50 and the boots; some, \$95 and the boots; and others, \$100 and the boots. Which is correct?

16. What relation to me is my mother's brother-in-law's brother, provided he has but one brother?

17. Three men, travelling with their wives, came to a river which they wished to cross. There was but one boat, and but two could cross at one time; and, since the husbands were jealous, no woman could be with a man unless her own husband was present. In what manner did they get across the river.

METHOD.—Let A and wife go over, let A return, let B's and C's wives go over, A's wife returns, B and C go over, B and wife return, A and B go over, C's wife returns, and A's and B's wives go over, then C comes back for his wife.

Simple as this question may appear, it is found in the works of Alcuin who flourished a thousand years ago; hundreds of years before the art of printing was invented. —*Parke.*

18. Suppose it were possible for a man, in Cincinnati, to start on Sunday noon, when the sun is in the meridian, and travel westward with the sun, so that it might be in his meridian all the time. He would arrive at Cincinnati next day at noon. Now, it was Sunday noon when he started, it has been noon with him all the way around, and is Monday noon when he returns. The question is at what point did it change from Sunday noon to Monday noon?

19. Suppose a hare is 10 rods before a hound, and that the hound runs 10 rods while the hare runs 1 rod. Now when the hound has run the 10 rods, the hare has run 1 rod; hence they are now one rod apart, and when the hound has run that 1 rod, the

hare has run $\frac{1}{10}$ of a rod; hence they are now $\frac{1}{10}$ of a rod apart, and when the hound has run the $\frac{1}{10}$ of a rod they are $\frac{1}{100}$ of a rod apart; and in the same way it may be shown the hare is always $\frac{1}{10}$ of the previous distance ahead of the hound; hence the hound can never catch the hare. How is the contrary shown mathematically.

20. Think of any three numbers less than 10. Multiply the first by 2, and add 5 to the product. Multiply this sum by 5, and add the second number to the product. Multiply this last result by 10, and add the third number to the product; then subtract 250. Name the remainder, and I will name the numbers thought of, and in the order in which they were thought of.

Method.—The three digits composing this remainder will be the numbers thought of; and the order in which they were thought of will be the order of hundreds, tens, and units.

21. Write 24 with three equal figures, neither of them being 8.

Method.—$22 + 2 = 24$, or $3^3 - 3 = 24$.

22. Put down four marks, and then require a person to put down five more marks, and make ten.

Method.—The four marks are as represented in the margin; the five more, making ten, are placed as in the margin.

23. Which is the greater, and how much, six dozen dozen, or one-half a dozen dozen, or is there no difference between them?

24. Show what is wrong in the following reasoning:— $8 - 8$ equals $2 - 2$; dividing both these equals by $2 - 2$ and the result must be equal; $8 - 8$ divided by $2 - 2 = 4$, and $2 - 2$ divided by $2 - 2$

= 1; therefore, since the quotients of equals divided by equals, must be equal, 4 must be equal to 1.

25. A man has a triangular lot of land, the largest side being 136 rods, and each of the other sides 68 rods; required the value of the grass on it, at the rate of $10 an acre.

REMARK.—The "catch" in this is, that the sides given will form no triangle.

26. Says A to B, "Give me four weights, and I can weigh any number of pounds not exceeding 40." Required the weights and method of weighing.

ANSWER.—The weights are 1, 3, 9, and 27 pounds. In weighing we must put one or more in both scales, or some in one scale and some in the other; thus, 7 lbs. = 9 lbs. + 1 lb. — 3 lbs.

27. Mr. Frantz planted 13 trees in his garden, in such a manner that there were 12 rows and only 3 trees in each row. In what manner were they planted?

ANSWER.—They were in the form of a regular hexagon, having a tree in the centre, and one at the middle and extremity of each side.

28. A and B raised 749 bushels of potatoes on shares; A was to have $\frac{3}{7}$, and B $\frac{4}{7}$ of them. Before they were divided, however, since A had used 49 bushels B took 28 bushels from the heap, and then divided the remainder according to the above agreement. Was this division fair? if not, show how it should have been.

29. Two-thirds of six is nine, one-half of twelve is seven,
The half of five is four, and six is half of eleven.

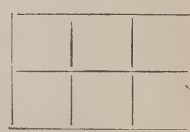
SOLUTION.—Two thirds of SIX is IX; the upper half of XII is VII; the half of FIVE is IV; and the upper half of XI is VI.

30. Does the top of a carriage wheel move faster than the bottom?

METHOD.—This seems absurd, but it is strictly true, as any one may satisfy himself in a moment, by setting up a stake by the side of a wheel, and move the wheel forward a few inches.

31. Supposing there are more persons in the world than anyone has hairs on his head, there must be, at least, two persons who have the same number of hairs on the head, to a hair. Show how this is.

32. Place 17 little sticks—matches, for instance—making 6 equal squares, as in the margin. Then remove 5 sticks, and leave 3 perfect squares of the same size.



33. Three persons own 51 quarts of rice, and have only two measures; one a 4 quart and the other a 7 quart measure. How shall they divide it into three equal parts?

METHOD.—Perhaps the easiest way is to give each one 17 quarts, which may be obtained thus: fill the 7 quart measure, empty this into the 4 quart measure, and there will be 3 quarts in the 7 quart measure, which added to two 7 quart measures, equals 17 quarts.

34. What four United States coins will amount to fifty-one cents?

ANSWER.—Two 25 ct. pieces and two half-cents.

35. How may the nine digits be arranged in a rectangular form, so that the sum of any row, whether horizontal, vertical, or diagonal, shall equal 15?

4	9	2
3	5	7
8	1	6

ANSWER.—As in the margin.

36. How may the first 16 digits be arranged, so that the sum of the vertical, and horizontal, and the two oblique rows, may equal 34?

ANSWER.—As in the margin.

1	16	11	6
13	4	7	10
8	9	14	3
12	5	2	15

37. In what manner may the first 25 digits be arranged, so that the sum of each row of five figures may be 65?

ANSWER.—As in the margin.

1	10	12	18	24
9	11	20	22	3
13	19	21	5	7
17	23	4	6	15
25	2	8	14	16

REMARK.—The above are called Magic Squares. They are very interesting, and have engaged the attention of some of our greatest mathematicians, among whom we may mention Leibnitz, Stifels, &c. The methods of arrangement given above are by no means the only ones that may be used. For the second problem, Frenicle, a French mathematician, has shown that there may be 878 different arrangements.

38. Take 10 pieces of money, lay them in a row, and require some one to put them together in heaps 2 in each, by passing each piece over 2 others.

METHOD.—Let the pieces be represented by the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Place 7 on 10, 5 on 2, 3 on 8, 1 on 4, and 9 on 6.

39. An old Jew took a diamond cross to a jeweller, to have the diamonds reset; and fearing that the jeweller might be dishonest, he counted the diamonds, and found that they numbered 7, in three different ways. Now the jeweller stole two diamonds, but arranged the remainder so that they counted 7 each way, as before. How was it done?

METHOD.—The form of the cross when left is represented by Fig. 1, and when returned by Fig. 2. It will be seen by the figures how the diamonds were counted by the old Jew, and how they were arranged by the jeweller, who "jewed" the Jew.

Fig. 1.	*Fig. 2.*
7	7
6	767
76567	5
4	4
3	3
2	2
1	1

40. Let a person select a number greater than 1 and not exceeding 10; I will add to it a number not exceeding 10, alternately with himself; and, although he has the advantage in selecting the number to start with, I will reach the even hundred first.

METHOD.—I make my additions so that the sums are, respectively, 12, 23, 34, 45, &c., to 89, when it is evident I can reach the hundred first. With one who does not mistrust the method, I need not run through the entire series, but merely aim for 89, or, when the secret of this is seen, for 78, then 67, &c.

41. Let a person think of any number on the dial-face of a watch; I will then point to various numbers, and at each he will silently add *one* to the number selected until he arrives at *twenty*, which he will announce aloud, and my pointer will be upon the number he selected.

METHOD.—I point promiscuously about the face of the watch until the eighth point, which should be upon "12;" and then pass regularly around, towards "1," pointing at "11," "10," "9," &c., until "twenty" is called, when, as may be easily shown, my pointer will be over the number selected.

42. Is there any difference between the results of the two following problems, and if so, what is it? If the half of 6 be 3, what will the fourth of 20 be? If 3 be the half of 6, what will be the fourth of 20?

43. A vessel with a crew of 30 men, half of whom were black, became short of provisions; and, fearing that unless half the crew were thrown overboard, all would perish, the captain proposed to the sailors to

stand upon deck in a row, and every ninth man be thrown overboard, until half the crew were destroyed. It so happened that the whites were saved. Required the order of arrangement.

Answer.—W. W. W. W. B. B. B. B. B. W. W. B. W. W. W. B. W. B. B. W. W. B. B. B. W. B. B. W. W. B.

This can easily be found by trial, using letters or figures to represent the men.

44. Think of a number, multiply it by 6, divide this product by 2, multiply by 4, divide by 3, add 40, divide by 4, subtract the number thought of, divide by 2, and the quotient is 5. Show why this is so.

45. If through passenger trains, running to and from Philadelphia and San Francisco daily, start at the same hour from each place (difference of longitude not being considered) and take the same time, six days, for the trip, how many through trains will the Pacific Express that leaves the San Francisco depot at 9 P. M., Sunday, have met when it reaches the Philadelphia depot?

46. A switch siding to a single track railroad is just long enough to clear a train of eight cars and a locomotive. How can two trains of sixteen cars and a locomotive each, going in opposite directions, pass each other at this siding, and each locomotive remain with and have the same relative position to its own train after as before passing?

47. Two hunters killed a deer, and sold it by the pound in the woods. They had no proper means of weighing it, but knowing their own weights—one 130 pounds and the other 190 pounds they placed a rail across a fence so that it balanced with one on each end. They then exchanged places, and the lighter man taking the deer in his lap, the rail again balanced. What was the weight of the deer?

VARIATIONS.

The doctrine of variations and combinations forms the basis of many forms of Lotteries, and of other calculations used in practical life. We shall commence with the simplest form of variations in which all the articles are taken at once and which is called

PERMUTATION.

To determine the number of permutations, commence with unity and multiply by the successive terms of the natural series 1, 2, 3, &c., until the highest multiplier shall express the number of individual things. The last product will indicate the number of possible changes.

Example 1. How many changes can be made in the arrangement of 5 grains of corn, all of different colors, laid in a row?

SOLUTION.—$1\times2\times3\times4\times5=120$, *Ans.*

This may seem improbable, the number being so great, but if there were but a single grain more, the possible changes would be 720; and another would extend the limit to 5040; and so onward in a constantly increasing ratio. The reason, however, will be obvious on a little scrutiny. If there were but one thing, as *a*, it would admit of but one position; but if two, as *a b*, it would admit of two positions, *ab*, *ba*. If three things, as *a b c*, then they will admit of $1\times2\times3=6$ changes, for the last two will admit of two variations, as *a b c*, *a c b*, and each of the three may successively be placed first, and two changes made to each of the others, so that $3\times2=6$, the number of possible changes. In the same way we may show that if there be four individual things, each one will be first in each of the six changes which the other three will undergo, and consequently, there will be 24 changes in all. In this way we might show that when there are 5 individual things, there will be 5 times as many changes as when there were but 4; and when 6, there will be 6 times as

many changes as when there are only 5; and so on ***ad infinitum***, according to the same law.

Example. 2. In how many ways may a family of 10 persons seat themselves differently at dinner?

ANSWER.—3628800.

When we consider that this would require a period of $9935\frac{55}{487}$ years, the mind is lost in astonishment. The story of the man who bought a horse at a farthing for the first nail in his shoe, a penny for the second, &c., is thrown into the shade; and we incline to doubt whether there is not some mistake; and yet on just such chances as one to all these do gamblers constantly risk their money!

Example 3. I have written the letters contained in the word NIMROD on 6 cards, being one letter on each, and having thrown them confusedly into a hat, I am offered $10 to draw the cards successively, so as to spell the name correctly. What is my chance of success worth?

ANSWER.—$1\frac{7}{18}$ cents.

48. Sold a horse for $56, and gained as much per cent. as the horse cost me. Required the cost.

RULE.—"Multiply the selling price by 100, and add 2500 to the product; of the sum extract the square root, and from the root subtract 50. The remainder will be the prime cost."

Horse sold for $56	Proof—
100	Cost $40
——	Per cent. 40
5600	——
+2500	Gain 16.00
——	Cost 40
√8100=90	——
—50	Sold for $56
——	——
Leaves $40 cost.	

Human ingenuity would perhaps fail to find a reason for the above rule, by the aid of common arithmetic merely, or to explain the steps satisfactorily to a learner. It seems to be without reason, and yet it will solve all questions involving a similar principle.

49. Suppose 2000 soldiers had been supplied with bread sufficient to last them 12 weeks, allowing each man 14 ozs. per day; but on examination they find 105 barrels, containing 200 lbs. each, wholly spoiled; what must be the allowance to each man, that the remainder may last them the contemplated time?

ANALYSIS, as follows:—1st. If one man ate 14 ozs. in a day, he would eat 7 times 14=98 ozs. in a week; and if he ate 98 ozs. in a week, he would eat 12X98=1176 ozs. in 12 weeks; and 2000 men would eat 2000X1176 ozs.=2352000 ozs. in 12 weeks.

105 barrels of 200 lbs. each=21000 lbs. destroyed; and 21000X16=336000 ozs., which deducted from the whole quantity 2352000 ozs. leaves 2016000 ozs. to be consumed. Then if 2000 men consume 2016000 ozs. in 12 weeks, 1 man will consume $\frac{2016000}{2000}$=1008, ozs.; and if 1 man consume 1008 ozs. in 12 weeks, he will consume $\frac{1008}{12}$=84 ozs. in 1 week, and $\frac{84}{7}$=12 ozs. in 1 day. *Ans.*

50. How may 100 be expressed with four nines?

ANSWER.—$99\frac{9}{9}$.

51. What three figures, multiplied by 4, will make precisely 5?

ANSWER.—$1\frac{1}{4}$, or 1.25.

52. Required to subtract 45 from 45 and leave 45 as a remainder.

SOLUTION.— $9+8+7+6+5+4+3+2+1=45$

$1+2+3+4+5+6+7+8+9=45$

$8+6+4+1+9+7+5+3+2=45$

53. From 6 take 9; from 9 take 10;

From 40 take 50, and 6 will remain!

SOLUTION.—

SIX	IX	XL
IX	X	L
S	I	X

The majority of these puzzles and problems being founded upon principles quite easily comprehended, it is not thought necessary to explain the principles of the puzzles nor solve the problems. It is hoped that they may prove a source of pleasure and profit.